

**IMITATION, LEARNING, AND COMMUNICATION:****CENTRAL OR POLARIZED PATTERNS IN COLLECTIVE ACTIONS****Ping Chen**

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**INTRODUCTION**

Neoclassical models in microeconomics often describe an atomized society, in which every individual makes his or her own decision based on individual independent preference without the communication and interaction with the fellow members within the same community.<sup>1</sup> Therefore, static economic theory cannot explain collective behavior and changes of social trends, such as fashion, popular name brand, political middling or polarization.

Physicists have been interested in collective phenomena caused by the success of ferromagnetic theory in explaining collective phenomena under thermodynamic equilibrium. The Ising fish model<sup>2</sup> and the public opinion model<sup>3</sup> represented the early efforts in developing complex dynamic theory of collective behavior. However, the human society is an open system, there is no ground to apply the technique of equilibrium statistical mechanics, especially, the Maxwell-type transition probability, to social behavior. Therefore, these pioneer models have been discussed in physics community, but received little attention among social scientists.

However, the development of nonequilibrium thermodynamics and theory of dissipative structure open new way to deal with complex dynamics in chemical systems and biological systems,<sup>4,5,6</sup> that have many similarities with the behavior of social systems. In this short article, we will introduce new transition probability into master equation from the consideration of socio-psychological mechanism. The model may shed light on social behavior such as fashion, public choice and political campaign.

## STOCHASTIC MODELS FOR IMITATION AND COLLECTIVE BEHAVIOR

Stochastic models are very useful in describing social processes.<sup>2,3,4,5,7</sup> To define basic concepts, we will start from the master equation and follow Haken's formulation of the Public Opinion Model.<sup>5</sup>

Let us consider the polarized situation in a society that only two kinds of opinions are subject to public choice. The two opinions are denoted by the symbols of plus and minus. The formation of an individual's opinion is influenced by the presence of the fellow community members with the same or the opposite opinion. We may use a transition probability to describe the changes of opinions.

We denote the transition probabilities by

$$p_{+-}(n_+, n_-) \text{ and } p_{-+}(n_+, n_-)$$

where  $n_+$  and  $n_-$  are the numbers of individual holding the corresponding opinions + and -, respectively;  $p_{+-}$  denotes the opinion changes from + to -, and  $p_{-+}$  denotes the opposite changes from - to +.

We also denote the probability distribution function by  $f(n_+, n_-; t)$ .

The master equation can be derived as following:

$$\begin{aligned} d f(n_+, n_-; t) / dt = & \\ & (n_+ + 1) p_{+-}[n_+ + 1, n_- - 1] f[n_+ + 1, n_- - 1; t] + \quad + \\ & (n_- + 1) p_{-+}[n_+ - 1, n_- + 1] f[n_+ - 1, n_- + 1; t] - \\ & \{ (n_+) p_{+-}[n_+, n_-] + (n_-) p_{-+}[n_+, n_-] \} f[n_+, n_-; t] \end{aligned} \quad (2.1)$$

We may simplify the equation by introducing new variables and parameters:

$$\text{Total population: } n = n_+ + n_-,$$

$$\text{Order Parameter: } q = (n_+ - n_-) / 2n$$

$$w_{+-}(q) = n_+ p_{+-}[n_+, n_-] = n(1/2 + q) P_{+-}(q)$$

$$w_{-+}(q) = n_- p_{-+}[n_+, n_-] = n(1/2 - q) P_{-+}(q)$$

Where,  $n$  is the total number of the community members,  $q$  measures the difference ratio and can be regarded as an order parameter,  $w_{+-}(q)$  and  $w_{-+}(q)$  are the new function describing the opinion change rate which is a function of order parameter  $q$ .

This equation can describe collective behavior such as formation of public opinion, imitation, fashion, and mass movement. The problem remains unsolved here is how to know the transition probabilities  $p_{+-}(n_+, n_-)$  and  $p_{-+}(n_+, n_-)$ . The choice of the form of transition probability is closely associated with the assumption of communication patterns in human behavior.

## ISING MODEL OF COLLECTIVE BEHAVIOR

The early attempt to solve the problem of human collective behavior was directly borrowed the formulation of transition probability of Ising Model under equilibrium phase transition,<sup>2,3,5</sup> that was developed to explain the phase transition from paramagnetic state to a ferromagnetic state on a magnetic lattice in equilibrium statistic mechanics.<sup>7</sup> Haken define the transition probability in analog of Ising model during equilibrium phase transition

$$\begin{aligned} p_{+-}[q] &= p_{+-}[n_+, n_-] = v \exp\{-(Iq+H)/Q\} = v \exp\{-(kq+h)\} \\ p_{-+}[q] &= p_{-+}[n_+, n_-] = v \exp\{+(Iq+H)/Q\} = v \exp\{+(kq+h)\} \end{aligned} \quad (3.1)$$

Where,

**I** is a measure of the strength of adaptation to neighbors or interaction constant. One could imagine that the interaction constant is low in an individual culture but high in an collective society.

**H** is a preference parameter or social field ( $H > 0$  means that opinion + is preferred to -). We may label liberal trend as +, and conservative trend as -. The corresponding variable of H in physics is the magnetic field. We may explain the social field as the results of propaganda, advertising, or political campaign.

**Q** or  $1/k$  is collective climate parameter or social temperature corresponding to  $kT$  in thermodynamics where  $k$  is the Boltzmann constant and  $T$  is the absolute temperature in physics. The social climate is warm (with high **Q** or low  $k$ ) when social stress or tension is low and tolerance allow diversified public opinion.

**v** is the frequency of flipping process.

For mathematical simplicity, here only show the cases of  $h=0$  when there are no centralized propaganda, and other means to form a dominating social opinion.

Master Equation (2.1) can be transformed into a partial differential equation. Under this simplification, the master equation has two kind of steady-state solutions [see Fig.1]. The analytic solution of steady state is:

$$F_{St}(q) = c \exp\{2 [K1(y)/K2(y)] dy\}/K2(q) \quad (3.2)$$

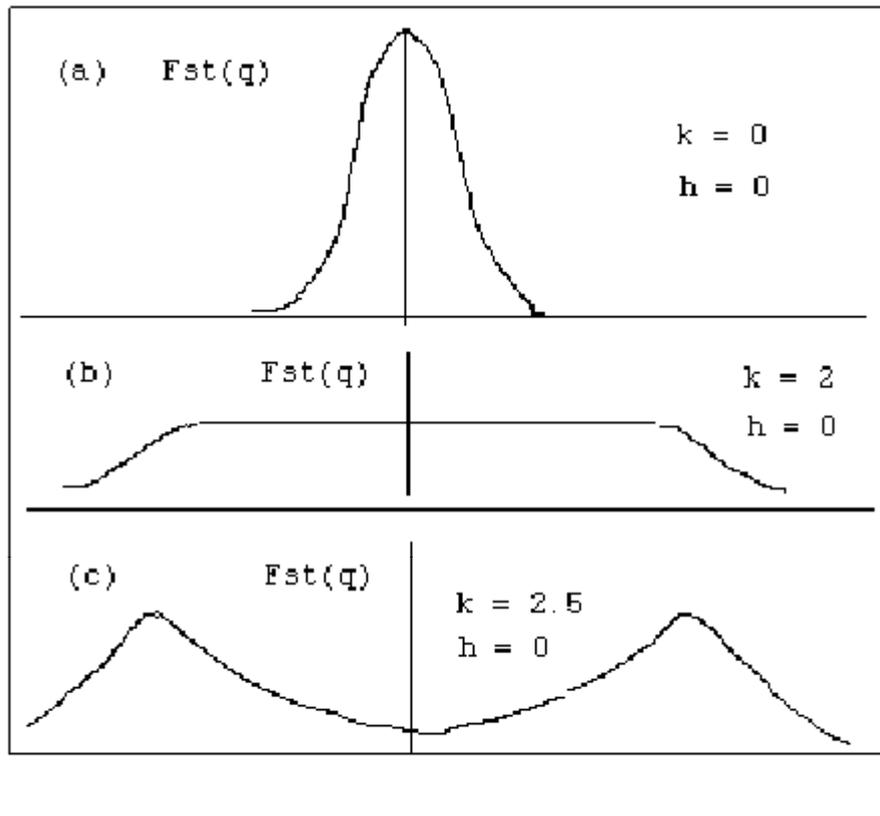


Fig.1. The steady state of probability distribution function in the Ising Model of Collective Behavior with  $h=0$ .

(a) Centered distribution with  $k=0$ . Middling behavior under high social temperature and weak interaction in societies.

(b) Marginal distribution at the phase transition with  $k=2$ . Behavioral phase transition occurs between middling and polarization in collective behavior.

(c) Polarized distribution with  $k=2.5$ . Strongly dependent behavior under low social temperature and strong group interactions in societies.

(1). The first kind of steady solution is the centered distribution when frequent changes of opinion occur. It happens under high social temperature and weak interpersonal interaction. For example, the case (a) (with  $k = 0$ ) may imply an individualistic atomized society;

(2). The second kind of steady solution is the polarized distribution that describes polarization in public opinions by strong neighbor interaction under low social temperature and strong group interaction. Say, the case (c) (with  $k > 2.5$  and  $h = 0$ ) represents the polarization in dense population society with collective culture

(3) If we consider the influence of mass media or advertising, then we have a social magnetic field  $H$ . When social field  $H > 0$ , or  $H < 0$ , the peak distribution may shift to right or left, by strong mass media or social trend.

Although Ising model of collective behavior gives some vivid qualitative pictures of social collective phenomena, its mathematical formulation is only an intellectual game that has not been taken seriously by social scientists.

There are several problems of using the Ising model in describing social phenomena. The first objection is that society is not an closed system. Therefore, the equilibrium thermodynamics and statistical mechanics cannot be applied to social systems. Then, the transition probability of Ising model becomes groundless in human systems. The second objection is that the social temperature  $Q$  and social field  $H$  have no measurable definition, therefore they are not observational indicators.

To overcome the difficulties of Ising model, we have to give up thermodynamic transition probability in equilibrium systems. We may find some plausible arguments to formulate the transition probability functions.

### **CHEN's SOCIO-PSYCHOLOGICAL MODEL OF COLLECTIVE CHOICE**

Consider a socio-psychological process with simple interaction relation.<sup>8</sup>

$$\begin{aligned} w[n_+, n_-] &= a_1 n_+ + b_1 n_+ n_- \\ w[n_-, n_+] &= a_2 n_- + b_2 n_+ n_- \end{aligned} \tag{4.1}$$

This transition probability has simple explanation: the rates of changes in personal opinion depend both on the population size holding the same opinion (as shown in the first term in the right hand side of equation) and the communication and interaction between the people holding opposite opinions. The outcome of collective choice is the result of the balance between individualistic orientation observed from independent choice and social pressure determined by the strength of communication and interaction. In other words, it depends on the relative magnitude of  $a$  and  $b$ .

Using new transition probability (4.1), we may solve the master equation (2.1) and have two kinds of steady state solutions [see Fig.2].<sup>9,10</sup>

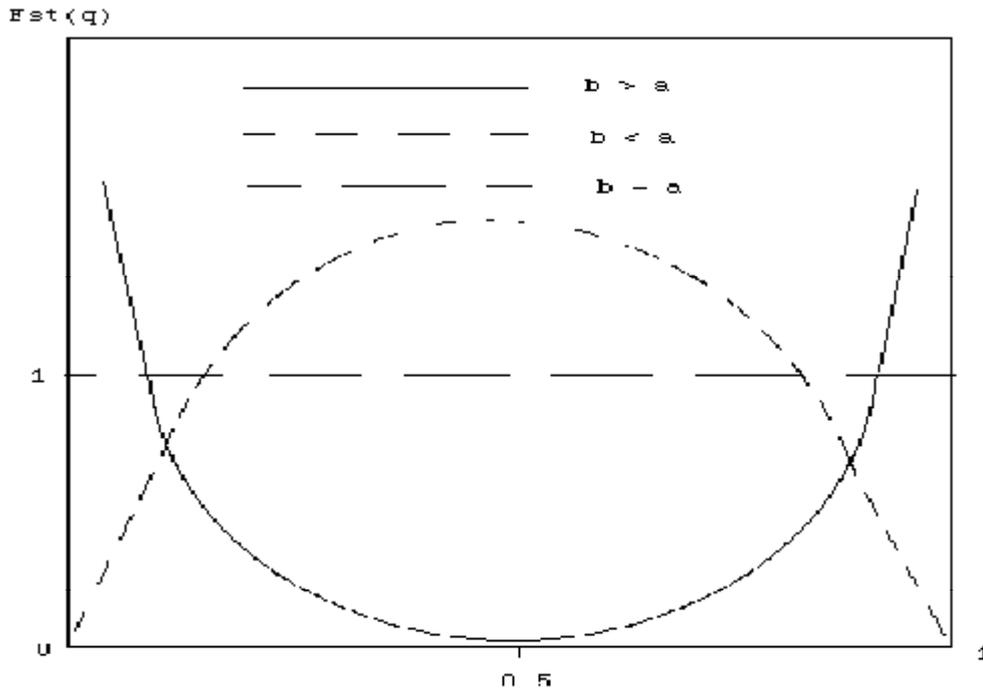


Fig. 2. The steady state of probability distribution function in socio-psychological model of collective choice.

(a) Centered distribution with  $b < a$  (denoted by short dashed curve). It happens when independent decision rooted in individualistic orientation overcomes social pressure through mutual communication.

(b) Horizontal flat distribution with  $b = a$  (denoted by long dashed line.) Marginal case when individualistic orientation balances the social pressure.

(c) Polarized distribution with  $b > a$  (denoted by solid line). It occurs when social pressure through mutual communication is stronger than independent judgment.

(1) When  $b < a$ , we have central distribution without polarization. The middling implies that independent decision overcomes the influence of social pressure in social systems. This phenomenon is quite familiar in the moderate social atmosphere of developed countries where the middle class plays a dominating role and middling attitude prevails in individualistic and pluralistic societies.

(2) When  $b > a$ , we have polarized distribution. It means that personal decision is much influenced by the social opinion. This situation is often observed in religious war and polarized revolution or mass movements. It may often cause by extremely poverty and social injustice in developing countries, when societies are polarized by racial, cultural, economic, and political polarization and confrontation in non-democratic social systems.

From above simple model and observation, we may conclude that individualistic culture is more likely to avoid polarized extremeness or protracted confrontation. The progress of

individualism may increase the opportunity to achieve a compromising attitude in human societies.

## POTENTIAL APPLICATIONS OF COLLECTIVE CHOICE

In this short paper, we only present the stochastic approach in addressing the collective behavior in human society. It is a modified version of Ising model by considering the interplay between individualistic orientation and social pressure through communication and interaction. We may also have the deterministic approach, which modifies the conventional competition model in theoretical biology by introducing learning behavior and cultural pattern.<sup>11</sup> Another interesting phenomena is nonlinear feedback control in human behavior. The time-delay and over-reaction may cause chaotic behavior because of bounded human rationality. This situation may happen when individual behavior is governed by collective trend and reacted to the deviations from the trend.<sup>12</sup> To explore the role of learning and communication in collective action and control behavior in complex human systems is an exciting new field. We have tremendous job to do. A few interesting problems are listed here for further development:

Theoretical biology: modeling learning behavior in prey-predator and competition models.

Sociology: modeling information diffusion, fashion and immigration.

Political Science: modeling class struggle, arm race, voting, political campaign, social trend, and differentiation.

Economy: modeling staged economic growth, fashion switching, risk-loving and risk-aversion competition, marketing and advertising strategy.

## CONCLUSION

Nonequilibrium physics opens new way to understand social evolution. Stochastic model with nonlinear transition probability is capable to describe the collective choice and social changes. Deterministic model of competition, learning, and nonlinear feedback control are also applicable in explaining social behavior.

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