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Microfoundations of macroeconomic fluctuations and the laws of probability theory: the principle of large numbers versus rational expectations arbitrage

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Abstract

The principle of large numbers shows that the relative deviation for a macro system with N independent elements is of the order of $1/\sqrt{N}$. Lucas' approach to a microfoundations for macroeconomic fluctuations is thus not capable of explaining the magnitude of observed macroeconomic fluctuations. Arbitrage activity would largely eliminate correlations created by rational expectations among economic agents when they face counter movements in relative prices. The complex nature of many-body problems and the statistical feature of aggregate indexes cannot be ignored in a micro–macro modeling. Intermediate structures, such as financial markets and industrial organizations, are more important than households and firms in generating business cycles. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The appropriate model for business cycle fluctuations with uneven growth trends is still an open issue in macroeconomics. There are two conflicting fundamental approaches to business cycle theory: the exogenous-shocks-equilibrium school (originating with Frisch, 1933) and the endogenous-cycles-disequilibrium school (originating with Samuelson, 1939).

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Lucas (1972) call for a microfoundations of macroeconomics based on the exogenous-shocks-equilibrium approach has had a strong impact on business cycle theory. Lucas emphasized two principles for equilibrium theory of business cycles. Optimal behavior should prevail at the micro level, and expectations should be formed rationally at the macro level. Lucas' approach implies that all unemployment and excess capacity is voluntary and optimal. The question this paper addresses is, "Can Lucas' theory explain the observed magnitude of business cycle fluctuations?" The answer is, "NO."

There are mainly two versions of optimization-equilibrium theory of business cycles. In economic thinking, the new classical school led by Lucas mainly considers monetary shocks while the real business cycle school (RBC) largely studies real (technological) shocks as the main source of external noise. In mathematical modeling, Lucas uses the island economy model of a stationary economy with many agents and the real business cycle school works with the representative agent model in computational simulation (Kydland and Prescott, 1982). In empirical studies, the new classical school follows the econometric convention of the log-linear (LL) detrending in regression analysis, while the real business cycle school develops the HP filter for separating smooth trend and fluctuating cycles in macroeconomic movements (Hodrick and Prescott, 1981). These two approaches to the microfoundations modeling of macroeconomic movements, however, have common problems: they ignore two fundamental issues, which are central to understanding the relationship between micro- and macro-dynamics in any system. Both Lucas and the RBC theory treat an essentially many-body problem as a one-body problem, and fail to appreciate the statistical nature of business fluctuations of prices and outputs that is changing over time.

It is a fundamental principle of empirical sciences that theory cannot be divorced from measurement. A natural measure of the fluctuation of positive data series (such as work hours and output in macroeconomic data) is the *relative deviation*, the ratio of the standard deviation of the series to its mean. According to the law of large numbers and the central limit theorem, the relative deviation is in the order of $1/\sqrt{N}$ for a system with N independent elements. We will call this general rule The Principle of Large Numbers.[2] This principle immediately calls into question the idea of explaining macroeconomic fluctuations as the aggregation of microeconomic fluctuations. The number of households and firms in the US economy is so large that the aggregation of microeconomic variations will produce a relative deviation several orders of magnitude smaller than observed macroeconomic fluctuations.

This article focuses on Lucas (1972) model of an island economy (the LMI model) as the benchmark model of the microfoundations approach in business cycle theory and gives only a brief discussion of the RBC model with a representative agent. In modeling stochastic processes, we extend our scope from statistics theory with stationary probability distribution (i.e. the i.i.d. model in econometrics) to probability theory with non-stationary probability distribution (here we use the linear birth–death process) for understanding macroeconomic fluctuations with growth, so that our empirical analysis can be applied to both the new classical model and the RBC model of macroeconomic fluctuations.

[2] The pattern of $1/\sqrt{N}$ is well known in physics and biology literature under various technical terms, such as the relative magnitude of fluctuation (Reif, 1964), the RMS relative fluctuation about the mean (May, 1974), and the fractional deviation (Reichl, 1999). The terms "relative deviation," "Principle of Large Numbers," and "positive variable" are used by the author to make them easier for general readers to understand the points at issue.

In the next two sections of this paper, we will show that the principle of large numbers is valid for stationary stochastic process like the system of the LMI model as well as for the linear stochastic process of growth studied in the RBC literature (Kydland and Prescott, 1990). There is little empirical evidence in favor of a microfoundation explanation of fluctuations in the US output and employment.

In Section 4 of the paper, we will discuss some theoretical issues raised by the LMI model. We will argue that, contrary to the claims of the LMI model, a rational expectations mechanism cannot be expected to generate perfectly correlated behavior among intelligent agents when they have market information and arbitrage opportunities. Certain fundamental factors underlying market movements, such as unequal distribution, economic complexity, and multiple time-scales, cannot be ignored in business cycle theory, as the LMI theory seems to claim.

The equilibrium framework based on microfoundations, rational expectations, and efficient markets is therefore not capable of providing a consistent explanation of business cycles. We must consider other alternatives, including the idea that the macroeconomy is undergoing chaotic deterministic dynamics and the idea that structures intermediate between micro (households and firms) and the macro economy play a crucial role in business cycle fluctuations.

2. Some statistical backgrounds

In this section, some statistical background is reviewed for discussing micro–macro relation in business fluctuations. The concept of relative deviation for positive random variables is analyzed for stationary and non-stationary stochastic process. The pattern of $1/\sqrt{N}$ emerges when N is large.

2.1. The relative deviation of a positive random variable

A basic measure of the fluctuation of a random variable X with a finite mean μ and higher moments is the *relative deviation, ψ,* is defined as the ratio of its standard deviation to its mean,

$$\psi = \frac{\sigma}{\mu} = \frac{\sqrt{\langle (X - \langle X \rangle)^2 \rangle}}{\langle X \rangle} \qquad (1)$$

where $\langle X \rangle$ represents the expectation of the variable X. The relative deviation provides a useful measure of the order of fluctuations, which is valid when the mean is not zero. This is certainly true for a *positive* variable. There is a wide class of positive variables in physics and social sciences, such as density, energy, population, output and working hours. Their values are non-negative so that their means are always greater than zero.

Consider some examples in statistics. The relative deviation for a point distribution is zero, for a two-point distribution is one, and for the uniform distribution over the unit interval, $\psi_{\text{uniform}} = 1/\sqrt{3} \approx 0.577$. About 99.7% of the Gaussian distribution falls within the range $(\mu - 3\sigma, \mu + 3\sigma)$. In order to regard a Gaussian random variable as a positive variable, therefore, we must have $(\mu - 3\sigma) > 0$, so we have

$$0 < \psi_{\text{Gaussian}+} = \frac{\sigma}{\mu} < \frac{1}{3} \tag{2}$$

2.2. The relative deviation in a system of two positive variables

Let us consider a system S_2 with only two variables X and Y. Its covariance and correlation coefficient are:

$$\text{COV}(X, Y) = \langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle = \langle XY \rangle - \langle X \rangle \langle Y \rangle$$

$$\rho = \frac{\text{COV}(X, Y)}{\sigma_x \sigma_y}$$

where σ_x and σ_y is the standard deviation for X and Y, respectively.

We can calculate correlation coefficients for two extreme cases. When X and Y are independent variables, we have $\langle XY \rangle = \langle X \rangle \langle Y \rangle$, and $\rho = 0$. When X and Y are linear dependent, then, X, Y are perfectly correlated and $\rho = 1$. If we further assume that X and Y have the same mean μ and variance σ^2 for simplicity, then, X and Y must be identical to have perfect correlation. These facts are useful in calculating the relative deviation of the system S_2. We have

$$\langle S_2 \rangle = 2\mu, \quad \langle S_2^2 \rangle = \langle X^2 + Y^2 + 2XY \rangle = 2\langle X^2 \rangle + 2\langle XY \rangle = 2\sigma^2 + 2\mu^2 + 2\langle XY \rangle$$

$$\text{VAR}[S_2] = \langle S_2^2 \rangle - \langle S_2 \rangle^2 = \langle S^2 \rangle - 4\mu^2 = 2\sigma^2 - 2\mu^2 + 2\langle XY \rangle$$

$$\langle XY \rangle = \langle X \rangle^2 = \mu^2 \quad \text{when } X, Y \text{ are independent}$$

$$\langle XY \rangle = \langle X^2 \rangle = \sigma^2 + \mu^2 \quad \text{when } X, Y \text{ are identical}$$

For the case of independent X and Y, we have VAR $[S] = 2\sigma^2$, so that

$$\psi_2 = \frac{\sqrt{\text{VAR}[S_2]}}{\langle S_2 \rangle} = \frac{1}{\sqrt{2}} \frac{\sigma}{\mu} = \frac{\psi}{\sqrt{2}} \quad \text{when } X \text{ and } Y \text{ are independent} \tag{3a}$$

For the case of identical X and Y, we have VAR $[S_2] = 4\sigma^2$, so that

$$\psi_2 = \frac{\sqrt{\text{VAR}[S_2]}}{\langle S_2 \rangle} = \frac{\sigma}{\mu} = \psi \tag{3b}$$

We can see that the relative deviation of a macro system with two elements is smallest with independent elements and largest with perfectly correlated elements. This result is perceivable since two independent fluctuations will partially cancel out each other. We can easily generate this result to a system with more elements.

2.3. The relative deviation in a system of many positive variables

Now, consider a stationary macro system with N identical positive elements, with values X_i, where $i = 1, 2, \ldots, N$. The sum describing the macro system is $S_N = X_1 + X_2 + \cdots + X_N$.

We assume that fluctuations in each positive variable follow an identical distribution, with mean μ and standard deviation σ. According to the law of large numbers in probability theory, the mean of the macro system is $N\mu$. Based on the central limit theorem, the variance of the macro system is $N\sigma^2$. Therefore, the relative deviation r for the macro system is

$$\Psi_{\mathrm{ST}} = \frac{\sigma\sqrt{N}}{N\mu} = \frac{\psi}{\sqrt{N}} \sim \frac{1}{\sqrt{N}} \tag{4a}$$

From Eq. (2), we have

$$0 < \psi = \frac{\sigma}{\mu} < \frac{1}{3} \tag{4b}$$

Thus, ψ is a finite constant, which is less than one. Therefore, the relative deviation of aggregated fluctuations with N positive independent elements must be on the order of $1/\sqrt{N}$, which can be applied when correlations among system elements are very weak.

The variance of the macro system will be much larger, however, when the correlations among micro elements are not near zero. Similar to the calculation in Section 2.2, we may easily calculate the extreme case of perfect correlations.

$$\langle S\rangle = N\mu$$

$$\langle S^2\rangle = N^2(\sigma^2 + \mu^2)$$

$$\mathrm{VAR}[S] = \langle S^2\rangle - \langle S\rangle^2 = N^2\sigma^2$$

Because the variance of the macro system with N perfect correlated elements is on the order of $N^2\sigma^2$, then its relative deviation has the same magnitude of the relative deviation of its micro variable.

$$\Psi_{\mathrm{ST(pc)}} = \frac{\sigma}{m} = \psi \quad \text{when } X_i, X_j \text{ are identical} \tag{5}$$

Clearly, the pattern of $1/\sqrt{N}$ can only apply to a system, whose elements are close to statistically independent.

2.4. *A non-stationary stochastic process*

If we apply the static principle of large numbers to macroeconomic data, we face the theoretical problem that a growing economy is not a stationary process. Let us therefore consider a finite-state stochastic process to represent internal fluctuations in a growth process. We assume the existence of a population of individuals. The case of exponential growth can be treated as a linear birth–death process with a constant growth rate r. The probability of a state n occurring at time t is denoted $P(n, t)$. Its rate of change with respect to time t is proportional to the population size n with a constant birth rate b, a death rate d, respectively:

$$\frac{\partial P(n,t)}{\partial t} = b(n-1)P(n-1,t) + d(n+1)P(n+1,t) - (b+d)nP(n,t) \tag{6}$$

Suppose the system is in the state of n at time t. There are four possibilities for a change in the proportion of subsystems in state n: a subsystem could move from state $(n-1)$ to

n through a birth process, or from state $(n+1)$ to n through a death process. Similarly a system could exit state n to $(n-1)$ through a death process or out of state n to $(n+1)$ through a birth process. We can calculate the mean and variance of the distribution over n once we have the analytical solution of $P(n, t)$ (Reichl, 1998):

$$\mu_{\mathrm{BD}} = \langle n(t)\rangle = \sum_n nP(n,t) = N_0 \mathrm{e}^{rt} \tag{7a}$$

$$\sigma_{\mathrm{BD}} = \langle n(t) - \langle n\rangle\rangle^2 = \sum_n (n - \langle n\rangle)^2 P(n,t) = N_0 \left(\frac{b+d}{b-d}\right) \mathrm{e}^{2rt}(1-\mathrm{e}^{-rt}) \tag{7b}$$

Here, N_0 is the initial population of the system, and the growth rate $r = b - d$.

The relative deviation of this system is

$$\Psi_{\mathrm{BD}} = \frac{\sigma_{\mathrm{BD}}}{\mu_{\mathrm{BD}}} = \frac{\sqrt{((b+d)/r)(1-\mathrm{e}^{-rt})}}{\sqrt{N_0}} \Rightarrow \frac{\psi_r}{\sqrt{N_0}} \sim \frac{1}{\sqrt{N}} \tag{8}$$

Their mean follows the same law of motion as a deterministic constant growth process. Comparing Eq. (8) with Eq. (4a), the non-stationary stochastic model of growth introduces the new standard deviation σ_{BD}. But this factor cannot change the general conclusion that the relative deviation grows inversely with the square root of N.

Consider a reasonable case of an annual growth rate $r = 4\%$, and $t = 20$ years. We have $\psi_r = 0.7421$ if $d = 0$, or $\psi_r = 1.0494$ if $d = 0.02$ and $b = 0.06$. These figures show that the principle of large numbers is also applicable for a linear stochastic system with growth.

2.5. *Implicit number of degrees of freedom and potential relative deviation*

The principle of large numbers suggests two useful statistics for the empirical analysis of micro–macro relations.

In a decentralized market economy, households, workers, and firms make their own economic decision. As a first approximation, we may consider a macroeconomic system consists of N independent agents, where N is the number of economic units. We define the *implicit number of degrees of freedom, N^**, as the number of independent micro positive variables that would produce the observed relative deviation of the macro variable, given the observed mean and standard deviation in a macroeconomic indicator:

$$N^* = \frac{1}{\Psi^2_{\mathrm{macro}}} = \frac{\mu^2_{\mathrm{macro}}}{\sigma^2_{\mathrm{macro}}} \tag{9}$$

Thus, the implicit number of degrees of freedom tries to back out the number of independent micro level processes that could produce the observed magnitude of macro level fluctuations.

Alternatively, if we know the actual number N_{micro} of microscopic elements in a macroscopic system, we can estimate the *potential relative deviation Ψ^** as the relative deviation we would expect to see in the macro fluctuations on the basis of the principle of large numbers:

$$\Psi^* = \frac{1}{\sqrt{N_{\mathrm{micro}}}} \tag{10}$$

We should keep in mind that the implicit number of degrees of freedom N^* is not an exact figure but an order of magnitude, since we do not have a uniform relative deviation from various sources of micro data. We use these figures to infer the effective number of elements in a micro–macro statistical model. We take out the constant in our evaluation of relative deviation for simplicity, since we are only interested in a rough estimation of the possible order of magnitude.

In empirical analysis, we may find the situation where the implied number from empirical fluctuations may be much smaller than the actual number of elements. There are two possible scenarios: If the difference between implied and actual number is not very large, it indicates that the system elements are weakly correlated. The principle of large numbers is still useful to provide an upper bound of number of elements. If the difference is very large, then we should consider structural analysis in micro–macro relation. The underlying dynamics could be a mixed process with deterministic and stochastic patterns. We will see both possibilities in analyzing the US macro data.

3. The empirical order of relative deviations and theoretical implications for business cycles

Based on the above discussion, we need to examine the empirical order of relative deviation in the US economy and discuss potential sources of the observed business fluctuations.

3.1. Detrending methods and observing windows

A salient character of most macroeconomic time series is their uneven growth trend (Fig. 1). Any practical measurement of business cycles must be based on a specific detrending method so that we can identify some regularity in a nonstationary time series. In the econometric literature, we find three such methods, which result in different estimated

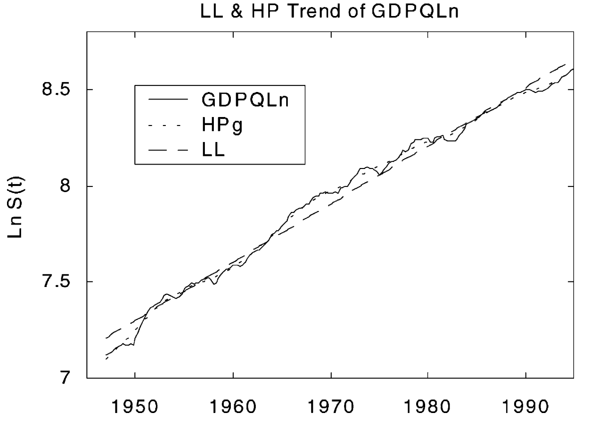

Fig. 1. The GDPQLn quarterly time series with LL and HP trends. The solid curve is the original logarithmic GDPQ time series; the dotted curve is the HP smooth trend; the dashed line is the log-linear (LL) trend.

Table 1
The relative deviation and implicit number of degrees of freedom for GDPQ using different detrending methods

Data	GDPQLn (HP)	GDPQLn (LL)	GDPQLn (ST)	GDPQLn (FD)
Ψ (%)	0.2	0.4	1.2	140
N^*	200000	70000	7000	0.5

The logarithmic GDPQ data is averaged over the period 1947–1995 with a moving time-window of 10 years. The implied numbers of degrees of freedom are rounded to one significant figure.

Table 2
The relative deviation and implicit number of degrees of freedom of GDPQ (HP) in different sub-periods

Data	1952	1960	1970	1980	1990	Mean (r)
Ψ (%)	0.31	0.15	0.23	0.24	0.13	0.21
N^*	100000	400000	200000	200000	600000	200000

magnitudes of the cyclic components of various macroeconomic time series. Log-linear (LL) detrending assumes the data follows a constant exponential trend over the whole period. The HP detrending estimates a nonlinear smooth trend (Hodrick and Prescott, 1981), which makes cyclic movements around the smooth trends fall within the frequency range of business cycles of about 4 years. First differencing (FD) allows the underlying trend rate of growth to change in each measurement period so that fluctuations tend to look like white noise. FD uses the shortest time window (just the time unit of a time series), and LL the longest (the full length of the time series). The effective window for the HP method depends on the smoothing parameter chosen which is often adjusted to the range of 4–8 years.

The FD method is not appropriate for the analysis of the micro–macro relation because the FD series has negative values. In addition, its relative deviation is larger than one, due to the noise-amplification inherent in first-differencing. Therefore, the FD reference is not proper for studying micro–macro relation.

For the US real GDP quarterly data, the average relative deviations implied by the three detrending methods plus the method assuming stationary (ST) are given in Table 1; the relative deviations under the HP method in different sub-periods are given in Table 2. The average relative deviations of other macroeconomic indicators in 1980 using the ST and the HP methods are given in Table 3.

From Table 1, we see that the HP trend produce the smallest relative deviation and the largest implied number from empirical data, which is closer to the empirical numbers of economic agents described in Section 3.2. Therefore, we take the HP trend as our standard

Table 3
The relative deviation and implicit number of degrees of freedom for several macro indexes under ST and HP methods

Method	Ψ (%) (N^*)			
	GDPQLn	GCQLn	GPIQLn	LBMNULn
ST	1.2 (7000)	1.4 (6000)	2.2 (2000)	1.1 (8000)
HP	0.22 (200000)	0.16 (400000)	1.3 (6000)	0.43 (50000)

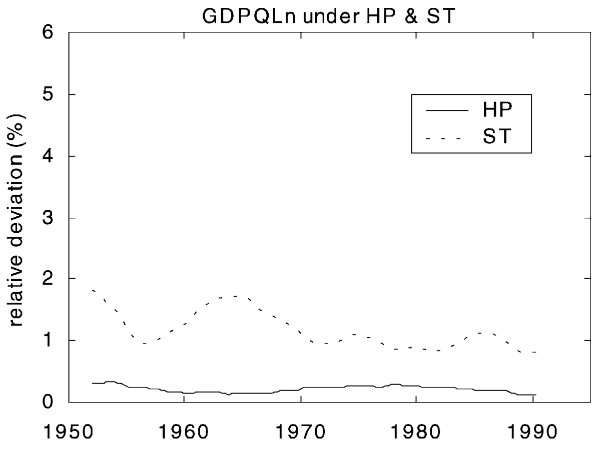

Fig. 2. The relative deviations of GDPQLn under the HP and ST references. The solid curve is the relative deviation under the HP reference; the dotted curve is under the ST. The width of moving time-window is 10 years.

reference, and use the ST method as the upper bound for the relative deviation for macroeconomic indicators. The choice of logarithmic GDPQ data is consistent with the econometric practice, which typically uses the logarithm of a macro series in analyzing business cycles.

From Fig. 2 and Table 2, we see that the relative deviation of the HP series is slowly changing over time with a long period about 25 years, from high of 0.3% during the Korea war in early 1950s to a low of 0.1% in early 1990s. Its implicit number of degrees of freedom lies between 100,000 and 600,000 in all sub-periods. There is no explosive tendency in the relative deviation. If we ignore the long-wave of the relative deviation, the linear birth–death model for exponential growth appears to be a first approximation for macroeconomic movements in the real GDP. Its implicit number of degrees of freedom on average is about 200,000.

Here GDPQ is the US real gross domestic product (US$) in 1987, GCQ is the real total consumption, GPIQ the real domestic investment, and LBMNU the hours of non-farm business. The data source is Citibank. The estimates of relative deviations are averages over the period between 1947 and 1995 with a moving time-window of 10 years.

Let us examine the HP results in Table 3. First, there is no evidence of $N = 1$, which is implied by the representative agent model. Second, the relative deviation of consumption is slightly less than that of GDP; but non-farm business hours and investment are much more volatile. Third, our measurement of the relative deviation reveals more information about the microstructure of the US economy than those papers in the RBC literature which measure the standard deviation of the HP cyclic components alone, without a comparison to the HP trend components (see, for instance, Kydland and Prescott, 1990). We will see this point in the following discussion.

3.2. Potential sources of business cycles

In Table 3, the relative deviations with HP detrending are roughly in a same order. The magnitude of their relative deviation is from 0.2 to 1.3%; its implicit number of degrees of

Table 4
Numbers of households and firms in 1980 of the US economy and corresponding potential relative deviations

Micro-agents	Households	Corporations[a]	Public companies
N	80.7 million	2.9 million	20000
Ψ^* (%)	0.01	0.05	0.7

[a] We count only those corporations with more than US$ 100,000 in assets.

freedom ranges from 6000 to about half a million. Can we associate these estimates with actual numbers observed in the US economy? According to the US Bureau of Census in 1980, the US civilian labor force was 106.9 millions, the relevant implied numbers and the potential relative deviations are given in Table 4.

From Tables 3 and 4, we can see that the actual relative deviation for the real GDP and non-farm business hours is 20–40 times larger than the potential relative deviation that could be generated by households; correspondingly, the implicit number of degrees of freedom estimated from real GDP and non-farm business hours are 400–2000 times less than the actual number of households. We can conclude that fluctuations at the household level are not capable of explaining the large observed relative deviations in real output and business hours. There is from this point of view little empirical evidence to support Lucas' theory, which tries to explain persistent unemployment, such as during the Great Depression, as a reflection of voluntary choices of workers.

The implicit numbers of degrees of freedom for non-farm business hours and real consumption are about 50,000 and half a million respectively. These figures suggest that it would be better to consider business organizations, such as labor unions and chain stores, rather than individual households and firms, as the basic subsystems driving fluctuations in modern market economies.

The potential relative deviation in real output and investment generated by the actual number of corporations in the US economy is 4–30 times smaller than the observed relative deviation. Correspondingly, the implicit numbers of degrees of freedom for fluctuations in real GDP and investment are 20–500 times smaller than the actual number of firms. This is a clue that large firms dominate at the micro level. There is weak evidence for the equilibrium picture of perfect competition at the firm level from macroeconomic data.

In the financial market, the potential relative deviation induced by the actual number of public companies is about half of the observed relative deviation in investment, while the implicit number of degrees of freedom from real investment fluctuations is roughly the same as the number of financial companies. This constitutes some evidence that fluctuations in the financial market have a much stronger impact than those in the output or labor market in generating business cycles.

4. Fundamental issues in microfoundations approach and rational expectations

In this section, we will discuss some fundamental issues concerning the statistical nature of micro–macro relationship within the equilibrium framework.

4.1. Essential difference between one-body and many-body problems

The representative agent model in the RBC literature is explicitly a one-agent model. So, $N = 1$, which is equivalent to assuming that all underlying microeconomic variables are perfectly correlated. This is drastically erroneous with the findings in Table 1.

A genuine model of a micro–macro relation should consider the statistical properties of a sum of elements. The LMI model appears at first to be a model with many agents but closer inspection reveals that it is a representative agent model in disguise. On the surface, the LMI posits N young producers and N old consumers in an overlapping generation framework in each time period. However, Lucas solves the optimization problem only for a representative agent, since he calculates the optimal consumption and labor supply not for each individual, but only for an average (expressed in per capita terms). The large number N in the Lucas model is an arbitrary parameter without economic consequences, which can be normalized to one as in the case of the indivisible labor model (Hansen, 1985).

To our knowledge, the microfoundations approach has not yet developed a full-fledged micro–macro model with many agents.

4.2. The statistical nature of economic information and market diversity

In the general equilibrium literature in microeconomics, all the agents engage in exchange with each other in a decentralized market. Therefore, price variation should be represented by a probability distribution. Even if all the agents within a homogeneous group have the same expectation, i.e. they agree on the mean value of future prices, the variance of actual exchange prices could not be zero in their exchange activities (McKenzie, 1987). The diversity of people's behavior is rooted in the degree of individual freedom, which is the essence of a decentralized market.

Strange as it might seem, the LMI model represents a centralized market without any freedom of individual choice. Its basic setting is quite simple: Exchanges occur at two separate markets in order to introduce a fluctuation in relative price. The young households are allocated stochastically, a fraction ϕ going to market A and fraction $(1 - \phi)$ going to market B.[3] Each young household supplies n units of labor and produces n units of real output. Here, the individual choice of n is determined by the allocation variable ϕ and money growth rate χ; the price is a unique function of three systematic random variables: ϕ, χ, and m, which is the pre-transfer money supply to old consumers. In other words, the exchange price and individual labor supply of $2N$ agents at each period are completely controlled by three systematic shocks at the macro level. No degrees of freedom are introduced to represent variation at the individual level in the LMI model.

The function of a centralized market is apparent in the determination of aggregate output (and employment) in the LMI model. The possible states of individual output are only two (the number of separate markets) not N (the total number of young households). We can

[3] We made one mathematical simplification for the LMI model. Here, our allocation variable ϕ is $(\vartheta/2)$ in the original LMI formulation.

see this from Lucas' Eq. (11). (We use Lucas's original numbers to label the equations in the LMI model.)

$$Y_t = N\left\{\phi_t n\left(\frac{\chi_t}{\phi_t}\right) + (1-\phi_t) n\left(\frac{\chi_t}{1-\phi_t}\right)\right\} \tag{11}$$

The meaning of Eq. (11) is straightforward. The total number of workers is N. Exchanges occur at two separate markets. There are $N\phi$ young producers at market A and $N(1-\phi)$ at market B. At each period t, the labor supply in each market is uniquely determined by $n(\chi_t/\phi_t)$ and $n(\chi_t/(1-\phi_t))$, respectively. That is why the total output in market A is the total producer number $N\phi$ times the average output $n(\chi, \phi)$, not a summation over n as in Eq. (7). From the point of view of statistical mechanics, Eq. (11) has the incredible implication that the behavior of all economic agents in each market is perfectly correlated, exactly as in the RBC case!

More accurately speaking, the effective number in the LMI model is two, because market A and B behave like two robot armies of consumers. Lucas deliberately chosen this number, since the relative deviation for a two-point distribution is largest (1) among all possible distributions for a positive variable, as we pointed in Section 2.1. However, implied numbers from empirical macro data are much larger than two.

4.3. Intertemporal substitution, relative price movements, and arbitrage opportunities

How could the independent agents end up with a perfectly correlated behavior in each market like a robot army? The device proposed in the LMI model to achieve this magical result is the rational expectations hypothesis.

Lucas has described his picture of rational expectations in his introduction to *Studies in Business-Cycle Theory* (Lucas, 1981):

> "It became clear to me why Phelps had imagined an island economy, with traders scattered and short on useful, system wide information. It is exactly this feature that permits all producers simultaneously to believe they have gained relative to others as the consequence of a monetary shock."

Lucas believed that the rational expectations mechanism could amplify price fluctuations by inducing the same beliefs and hence the same behavior in many people simultaneously. Muth, the originator of the rational expectations concept, on the other hand, thought rational expectations would reduce price fluctuations through arbitrage (Muth, 1961).

> "Speculation with moderately well-informed price expectations reduces the variance of prices by spreading the effect of a market disturbance over several time periods, thereby allowing shocks partially to cancel one another out. Speculation is profitable, although no speculative opportunities remain."

We can see that the conflicting perceptions of rational expectations of Muth and Lucas raise a series of open questions on directions of relative price movements and the behavior of independent rational agents.

One critical question is how there could be any arbitrage opportunities when the great majority of economic agents in one market take the same orchestrated action as envisioned by the LMI model. Arbitrage is possible when the market is decentralized and commodity exchanges occur at different prices. We will argue that individual variability in transaction prices and arbitrage possibilities cannot be excluded from a market when the appropriate statistical framework is introduced into the LMI model.

Consider a simple scenario starting with a random shock in money supply or technology, which generates a systematic shock in output prices and wages. (Note that since the LMI model does not envision any price difference between labor and output—no profit motivation exists for producers in the model). Choose one of the two markets, say, market A. There are $N\phi_t$ identical agents having same rational expectations after a shock so that they all believe the current average wage is below the natural rate, and the future average price and average wage will rise to equilibrium in the next period (25 years later!).

Now we want to ask whether all these identical agents would make the same responses even if they share the same information (whether the information is "perfect" or "imperfect"). Our answer is NO if these rational agents are not price slaves but price setters in a general equilibrium market.

First, the labor or output price must be varied across economic agents. If we assume that the only output is food for human subsistence, then we will conclude that exchange must occur between agents since a cleared market has no inventory for each household. In a decentralized market, there is no mechanism that will lead to uniformity of transaction prices when exchanges occur between scattered agents. Therefore, the market price of labor or output must be a statistical average (with some variance) over the agents in a market. A price distribution with finite mean and non-zero variance would certainly occur when economic agents have unequal wealth and varying needs of leisure. The LMI model with identical agents should be analogous to the ideal gas model in statistical mechanics, in which all particles have the same mass and elasticity but different initial conditions in coordinates and speed. If all particles are assumed to have the same coordinates and speed, the N molecules become a concentrated point mass rather than a gas with $6N$ degrees of freedom. Its failure to consider individual degrees of freedom is a fundamental flaw in the LMI model, which is not, as a result, consistent with the original idea of general equilibrium in a decentralized market.

Second, transaction price variation must create arbitrage opportunities that would offset the intertemporal substitution effect under rational expectations. Let us consider a simple vacation scenario of relative price (wage and vacation price) movements initiated by a macroeconomic shock in the LMI model.

The time scale of the LMI model is a human generation, on the order of, say, 25 years. Suppose the current average wage is below the natural rate, so that all the agents agree that the optimal supply of labor is, say, 40% of normal working hours in this period. Can we imagine that *all the agents* would take a *synchronized* vacation during the first 15 years then work at the next 10 years as implied by the rational expectation hypothesis? No, the market would not function continuously if household behavior were so uniform. A more reasonable picture is that vacation plans would vary in such a way that the average work ratio over this period is 40%. The different time scales between labor contracts and monetary shocks create conflicting movements in relative prices and arbitrage opportunities for utility-maximizing agents.

If economic agents reach an agreement on an expected mean (or rational expectations), can we predict a net outcome (monetary neutrality or not) in relative price movements? The answer depends on certain unspecified structures in the LMI model. Lucas made strong claims concerning relative prices and general equilibrium in his analysis. In practice, Lucas had only one narrow type of relative price in mind, that is, the current and future wage. To achieve his goal of demonstrating monetary neutrality, Lucas abstracts from two important relative price movements in each period: the relative price between work and leisure at the supply-side of producers and the relative price between finished goods (say food) and leisure at the demand-side of consumers. Lucas implicitly assumed that movements in relative prices are not allowed within a period. The price mechanism in the LMI model is neither a partial equilibrium, nor general equilibrium at a single moment in time, but a self-centered intertemporal equilibrium for an individual atom in a market with a time unit of 25 years. This picture is hard to sustain as an explanation of business cycles in market economies.

Let us assume that only M households (M is large and $M < n\phi_t$) take their vacation in the first sub-period, say the first quarter. The large resulting demand shift would drive up the price of leisure goods, like airfare or vacation club prices. The increased cost of leisure would change the incentives of the rest of the households. So, $(N\phi_t - M)$ of the economic agents would have an incentive for inverse substitution by postponing vacation in this sub-period instead. Their arbitrage activities could offset the intertemporal substitution effect of the first group with M agents in the labor market. The net result will reduce or eliminate correlations between outputs among identical agents in market A.

Is there a neutrality of money in this scenario? The answer depends on the power balance of conflicting interests between the rich (who would be first to take their vacation when a recession begins) and the poor (who would be first to take the arbitrage opportunity when the relative price moves). Therefore, the main mechanism driving a decentralized market with an unequal distribution of income is conflicting interests, not common beliefs (Olson, 1965).

There is a symmetrical story for the $N(1 - \phi_t)$ agents in market B. In sum, the statistical nature of relative price movements cannot be ignored in a genuine micro–macro model with many independent agents.

4.4. Path variability and the Lucas critique

Some readers may contend that the above criticism is beyond the scope of the LMI model. No theoretical model can include every feature in nature. The question is if the theoretical model provides a consistent answer to the question it asked. If we take the LMI model as a starting base, can we reach a consistent explanation of business cycles?

The Lucas critique of conventional econometrics rightly pointed out that “any change in policy will systematically alter the structure of econometric models” (Lucas, 1976, 1981). However, a similar critique can also be applied to the microfoundations models in macroeconomic theory: Is the LMI model itself immune from Lucas’ own critique?

Suppose econometricians could fit a discrete-time model with rational expectations to quarterly data and believe economic agents will use this model to forecast its path for future periods. Then, we will see that any wave in relative price movements and arbitrage activities starting in the current quarter will systematically alter the structure of the econometric model and lead to deviations from the previously forecasted path in the next quarter. In other words, the LMI model suffers from the same flaws the Lucas critique identified in traditional macro econometric models.

Friedman once believed that speculators who based their trading strategy on the belief in the eventual emergence of equilibrium (i.e. rational or negative feedback strategy) always take money away from noise (i.e. irrational or positive feedback strategy) traders (Friedman, 1953). This is not true when arbitrage risk and dynamical complexity exist (see De Long et al., 1990; Chen, 1999). As the collapse of long-term capital has shown, arbitrage is risky when the future economic trends are unpredictable.

These two factors create a dilemma for the rational expectations research program. The more agents believe in the existence of a natural rate or rational expectations, the more they open up opportunities for arbitrage, and the smaller is the probability of success for maintaining the rational expectation co-movements. In other words, the rational expectations hypothesis is a self-defeating prophecy. Lucas once claimed that government policy was effective only when it was unexpected. Analogously, rational expectations cannot last very long when they mislead its believers!

4.5. Multiple time scales, information complexities, and the neutrality of money

The issue of "perfect" versus "imperfect" information is superficial, since there is no operational way to distinguish real from monetary disturbances within a short period. From the discussion in Section 3.1, there exist multiple relevant economic time scales, which are represented by various detrending methods. In real markets, globalization leads to 24 h trading in financial markets. Arbitrage activities are conducted in seconds in electronic exchange markets. The Federal Reserve adjusts the base interest rates on a weekly basis. Most macro indicators are published on a monthly or quarterly basis. These multiple time scales raise a fundamental problem for any simple discrete-time model. As Sims pointed out, the average period of a generation is about 20–30 years, while the period of the business cycle is in the range of 2–10 years, so how can we expect the overlapping generation model to be relevant to business cycle theory (Sims, 1986)?

In a world of economic complexity and multiple time scales, there is no role for the concept of "perfect information" or "all available information" in decision-making. In any scientific research, the operational issue is what is the relevant information available to answer a specific empirical question? Specifically, which variables are pertinent in theoretical modeling? What order, say, which moments of statistical data are sufficient in evaluating complexity and uncertainty? The rational expectations school assumes that rational agents will use all available information to form their rational expectations. But in practice, their scope is confined to a narrow band of available data.

Consider the debate over pertinent variables in financial analysis. The fundamental school mainly focuses on price information while the technical school also considers other information on quantity movements and market psychology. If traders ignored changes in price variation and trading volume as the equilibrium business cycle models assume, there would be no arbitrage activities, nor value-discovering mechanisms in market economies.

The economic market is like a democratic parliament. Even when they have all available information, economic agents rarely reach a consensus. If Ricardo had proposed his famous thought experiment of doubling all people's money holding as a piece of legislation, he would have had no chance to win a majority vote in the parliament, since the proposal would lead to a regressive subsidy in a democratic but unequal society (Chen, 1999).

Mathematically speaking, the rational expectations hypothesis is a simple model based on considering only the first moment of statistics. The LMI model replaces the theoretical price distribution of probability by a two-point distribution while the RBC model uses only one realization of a random trajectory. In financial analysis, 2nd to 4th moments are widely used. In fluid dynamics, up to seventh moments are studied in theory and experiments on turbulence. Is there reason to believe that information in macroeconomics is so much simpler than that in finance and physics?

If we recognize the complex nature of economic dynamics, we may reach a new understanding of the difficulty of understanding monetary neutrality.

First, the neutrality of money has no simple pattern under multiple time scales in economic dynamics. Monetary policy has a direct impact only on short-term interest rates but an indirect impact on medium and long-term interest rates.

Second, whether a monetary shock would produce purely monetary or real effects is not a simple question of people's belief in government policy, but a complex problem in the dynamics of economic structure. Lucas once acknowledged, "rational expectations are equivalent to the existence of a natural output rate" (Lucas, 1981). In the 1990s, the United States witnessed a period of high growth, low inflation, and a low unemployment rate, which is below the so-called "natural rate" or NAIRU (Galbraith, 1997). The debate on NAIRU or monetary neutrality has faded in the "new economy" because changes occurred in economic structure not in academic doctrine.

Finally, the increasing frequency and scale of financial crisis around the world reminds us that financial intermediation plays a more important role than household and firm behavior in generating business cycles and economic crisis.

5. Conclusion

Mathematics is not just a tool in economic theory. It is also a discipline in quantitative analysis. The principle of large numbers is a basic constraint in probability theory, statistical mechanics, and models of micro–macro fluctuations.

There is a striking asymmetry in the numbers of workers and organizations. This is the most visible characteristic of an industrial society. The new classical school raised an important question in the study of the relation between micro structure and macro fluctuations. Unfortunately, they did not uncover the main source of business cycles. The microfoundations models often ignore the essential difference between one-body and many-body problems and the statistic nature in modeling micro–macro dynamics. According to the principle of large numbers, there is little hope for a microfoundations model in the labor market, since the relative deviations of aggregate micro fluctuations are much smaller than those observed in macroeconomic indicators. The empirical and theoretical evidence indicates that the macroeconomic system can be better described by a three-level (micro–meso–macro) rather than a two-level (micro–macro) system (Holland, 1987). A further study of industrial organizations and financial intermediate may provide a better understanding of the observed magnitude of relative deviations from macro indicators.

Lucas claimed that his 1972 model is a "first" and "rigorously formulated" equilibrium model of business cycles. Lucas made a significant contribution in developing mathematical economics in business cycle theory. However, Lucas did not develop a quantitative model with probability distributions in prices and outputs, which can explain observed magnitude of business cycles. The harmonious picture of monetary neutrality abstracts away conflicting interests and arbitrage opportunities, which are essential for market competition and pricing mechanism. Theoretically speaking, the LMI model is not consistent with the original idea of general equilibrium with many goods, many agents, and interdependent changes in relative prices. The fundamental issue of economic complexity (including multiple time scales, structural variability, nonlinear interaction, and dynamic uncertainty) cannot be ignored in business cycle theory. Although we mainly discuss the LMI model in this article, readers may consider similar problems in other equilibrium models of microfoundations and rational expectations. From our analysis, the linear-equilibrium framework so far has not provided a consistent framework for business cycle theory (Chen, 1999).

If the new classical school did not shed much light on business cycle theory, can we draw some valuable lessons from their idealistic efforts? From our observation of large differences between the implied numbers from macro fluctuations and the actual numbers of economic agents and firms in the US economy, there are three possible alternatives for business cycle theory: nonlinear economic dynamics at the macro level (Chen, 1996); structural analysis of financial intermediate and economic institution; and macro foundations for micro behavior. We will discuss these approaches elsewhere.

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