Trends, Shocks, Persistent Cycles in Evolving Economy : Business Cycle Measurement in Time-Frequency Representation

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Abstract

One basic problem in business cycle studies is how to deal with nonstationary time series. Trend-cycle decomposition is critical for testing competing dynamic models, including deterministic and stochastic approaches in business cycle theory.

A new analytical tool of time-frequency analysis, based on the symmetry principle in frequency and time, is introduced for studies of business cycles. The Wigner-Gabor-Qian (WGQ) spectrogram shows a strong capability in revealing complex cycles from noisy and nonstationary time series.

Competing detrending methods, including the first difference (FD) and Hodrick-Prescott (HP) filter, are tested with the mixed case of cycles and noise. FD filter does not produce a consistent picture of business cycles. HP filter provides a good window for pattern recognition of business cycles.

Existence of stable characteristic frequencies from economic aggregates provide strong evidence of endogenous cycles and valuable information about structural changes. Economic behavior is more like an organism instead of random walks. Remarkable stability and resilience of market economy can be seen from the insignificance of the oil price shocks and the stock market crash. Surprising pattern changes occurred during wars, arm races, and the Reagan administration. Like microscopy for biology, nonstationary time series analysis opens a new space for business cycle studies and policy diagnostics.

The role of time scale and preferred reference from economic observation is discussed. Fundamental constraints for Friedman's rational arbitrageurs are re-examined from the view of information ambiguity and dynamic instability. Nonlinear economic dynamics offers a new perspective in empirical measurement and theoretical analysis.

JEL #C1, C2, C5, E3, N1.

Analyzing business cycles means neither more nor less than analyzing the economic process of the capitalist era.

> *Cycles are not, like tonsils, separable things that might be treated by themselves, but are, like the beat of the heart, of the essence of the organism that displays them.*
>
> *Joseph A. Schumpeter*
> *Business Cycles, A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*
> *McGraw-Hill, New York (1939).*

I. Introduction

An alternative title for econometric literature could be: "Business cycle measurement without model specification."

One basic difficulty in business cycle studies is that measurement is behind observation. We need analytical tools to characterize economic complexity. Hitherto, studies of business cycles are based on two alternative methods in time series analysis. Correlation analysis measure mean, variance, and correlations based on the covariance-stationary model of i.i.d. (the identical independent distribution) process in the time domain. The Fourier spectral analysis measures frequency and amplitude based on the cycle-stationary harmonic oscillations in the frequency domain. However, all these quantities are subject to changes in the time scale of business cycles. Real signals of economic movements contain both stochastic and deterministic components; therefore, we need new tools in time series analysis and business cycle modeling.

Economists realize the need to study nonlinearity and nonstationarity. There are two strategies to address the issue. One strategy is developing nonlinear and nonstationary versions of correlation analysis [Granger and Teräsvirta 1993]. Another strategy is developing nonlinear and nonstationary representations of spectral analysis [Chen 1993a]. We will focus on the second approach in this paper, since it is still in its infancy.

To address time-dependent phenomena, we introduce a new tool of time-frequency analysis which originated in quantum mechanics and acoustic physics [Wigner 1932, Gabor 1946]. A recent development in signal processing provides an efficient algorithm to calculate time-frequency distribution, which is a powerful tool to identify deterministic components from short and noisy signals [Qian and Chen 1994a, b, Chen and Qian 1993].

Most economic indicators have fluctuations around a growth trend. Different detrending methods lead to competing perspectives in business cycle theory. Two detrending methods are tested by time-frequency analysis: the first differencing (FD) and the Hodrick-Prescott (HP) smoothing filter. We find HP is much better than FD in revealing deterministic patterns from economic time series.

From a wide range of aggregate data, we find the existence of persistent cycles, in addition to color noise. Spectral analysis not only provides complementary evidence of "co-movements" of business fluctuations [Lucas 1981, Kydland and Prescott 1990], but also reveal distinctive patterns of frequency evolution. It is found that characteristic frequencies of business indicators are remarkably stable. Only minor changes occurred under such events, for example, the oil price shocks in 1973 and the stock market crash in 1987. Surprisingly, more significant changes happened during the Vietnam War and the Reagan administration. The time lag between frequency responses of different indicators provide important information about the propagation mechanism in the real economy. A new approach of economic diagnostic and policy evaluation can be developed quantitatively.

The new perspective of time-frequency analysis indicates fundamental barriers for Friedman's rational arbitrageurs against market disequilibrium. The role of time scale, observation reference, dynamical instability, and information ambiguity in studies of business cycle theory is discussed.

II. Time-Frequency Representation and Complex Economic Dynamics

It is known that the deterministic and the stochastic approach are complementary representations of dynamical systems. There are trade-offs in finite realizations of empirical signals. Which representation is useful in science is not a matter of philosophical debate, but a subject of empirical experiment.

Recent development of nonlinear economic dynamics demonstrates that business fluctuations can be explained by deterministic chaos [Benhabib 1992, Day and Chen 1993]. Standard tests of deterministic chaos are based on the phase space representation. The phase space approach has limited applications in empirical analysis, since a large number of data points and low level of noise is needed to calculate correlation dimension or construct the Poincáre section [Chen 1988, 1993a, b]. In contrast, time-frequency analysis has a much stronger power to deal with noisy data.

Time-frequency analysis is a powerful tool in distinguishing white noise and complex cycles. Complex cycles are nonlinear chaotic cycles with irregular amplitudes and sophisticated frequency patterns that are generalizations of linear harmonic cycles with regular amplitudes and well-defined frequencies.

(2.1) Time-Frequency Distribution and The Uncertainty Principle

The simplest time-frequency distribution is the short-time Fourier transform (STFT hereafter) by imposing a shifting time window in the conventional Fourier spectrum. STFT has poor resolution in the frequency domain caused by the finite square window in the time domain.

The Wiener-Khinchine theorem indicates that the autocorrelation and the power spectrum are Fourier pairs for a continuous time stationary stochastic process [Priestley 1981]. A natural generalization for the nonstationary process is introducing the instantaneous autocorrelation function $R_t(\tau)$ in the time-dependent power spectrum $P(t, \omega)$:

$$P(t,\omega) = \frac{1}{2\pi}\int R_t(\tau)\ \exp(-i\omega\tau)\,d\tau \qquad (2.1)$$

where the angular frequency $\omega = 2\pi f$.

Considering a symmetric time window, $R_t(\tau)$ can be replaced by the kernel function $s(t + \frac{\tau}{2})\, s^*(t - \frac{\tau}{2})$ to produce a time-dependent power spectrum called the Wigner distribution (WD) [Wigner 1932]:

$$WD(t,\omega) = \int S(t+\frac{\tau}{2})\, S^*(t-\frac{\tau}{2}) \exp(-i\omega\tau)\,d\tau \qquad (2.2)$$

An important development in time-frequency analysis is the Gabor expansion [Gabor 1946]. The best resolution in the frequency domain can be achieved by imposing a Gaussian window according to the uncertainty principle in signal processing [Gabor 1946, Papoulis 1977]:

$$\Delta t \ \Delta f \geq \frac{1}{4\pi} \qquad (2.3)$$

where the equality holds only for the Gaussian function.

Unfortunately, both the Wigner distribution and the Gabor expansion are un-orthogonal. The Wigner distribution is hard to calculate in continuous time because cross-interference terms are generated by non-orthogonal bases.

A synthesis of these two approaches (WGQ hereafter) leads to a good resolution and efficient algorithm [Qian 1992, Qian and Chen 1994a, b].[*] The Wigner distribution can be decomposed via the orthogonal-like Gabor expansion in discrete time and frequency. The localized symmetric base function has the form:

$$WD_b\,(t, \omega) = 2 \exp\{- [\,(t/\sigma)^2 + (\omega\sigma)^2\,]\,\} \qquad (2.4)$$

The time-frequency distribution series are constructed as approximations of the Wigner distribution.

$$TFDS_D(t,\omega) = \sum_{0}^{D} P_d\,(t,\omega) \qquad (2.5)$$

The zero-th order of time-frequency distribution series leads to STFT. The infinite order converges to the Wigner distribution. For applied analysis, 2nd or 3rd order is a good compromise in characterizing frequency representation without severe cross-term interference.

The WGQ representation in time-frequency analysis have important properties in physics and economics. The Wigner distribution ensures the conservation of energy density. This implies the conservation of variance in a time series analysis that is a key constraint in statistical analysis. The Gabor expansion catches periodic components under local observation. The time-

[*] The numerical algorithm has no specific name in the academic field. The originators call their approach the time-frequency distribution series from the view of mathematical formulation. The computer software is marketed by the National Instruments under the commercial name of Gabor spectrogram as a tool kit in the LabView System. The term WGQ representation is proposed by the author from the view of theoretical physics. Certainly, the author will take sole responsibility for this term. We will address this issue elsewhere.

frequency distribution series retain leading terms in the energy distribution. These features are critical in analyzing complex dynamics.

(2.2) Time-Frequency Analysis of Noise and Chaos

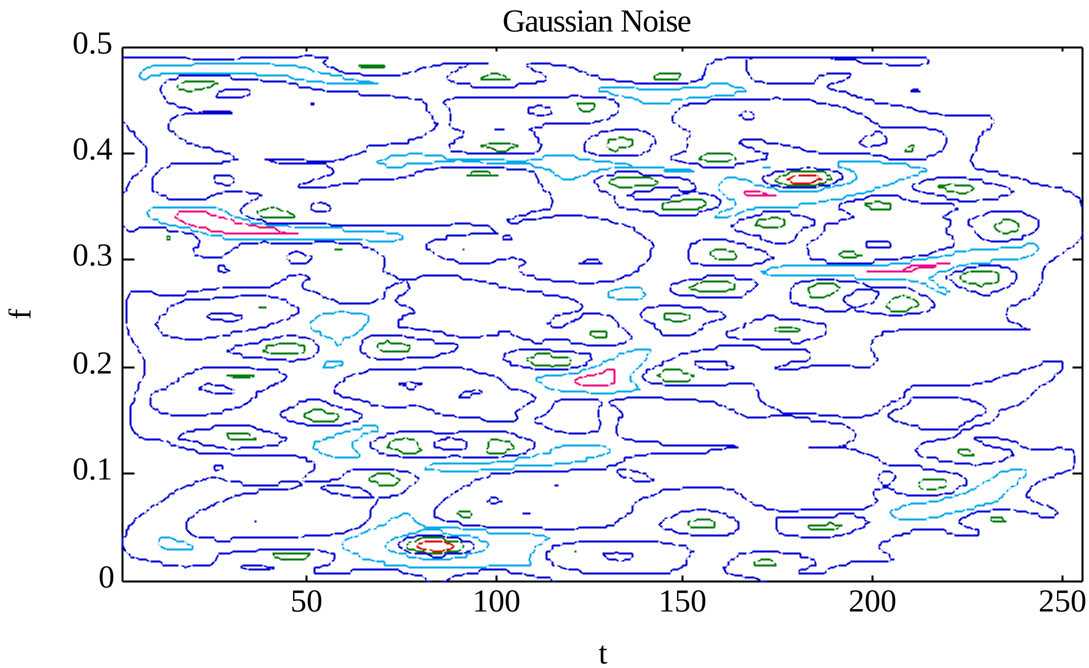
The development of nonlinear dynamics provides an alternative model of seemingly erratic movements: deterministic chaos, including white chaos (such as the logistic map and Henon map) in discrete time and color chaos (such as Rössler model) in continuous time [Hao 1990]. The "color" of continuous time chaos is characterized by its characteristic frequency f_c or characteristic period P_c observed in spectral analysis.

There are some limitations for spectral analysis in testing deterministic chaos. To avoid aliasing effect, the standard frequency window in spectral analysis is one half, or P_{min}, the lowest period observable, is two. The characteristic period and the characteristic frequency of white chaos is equal to one. Therefore, white chaos is outside the observational window. Spectral analysis alone is not sufficient to test the existence of color chaos; complementary measurements are needed [Chen 1993a]. For studies of business cycles, the discovery of a characteristic frequency of erratic time series provides essential information about the components of deterministic cycles, regardless of whether they are pure color chaos or mixed color noise.

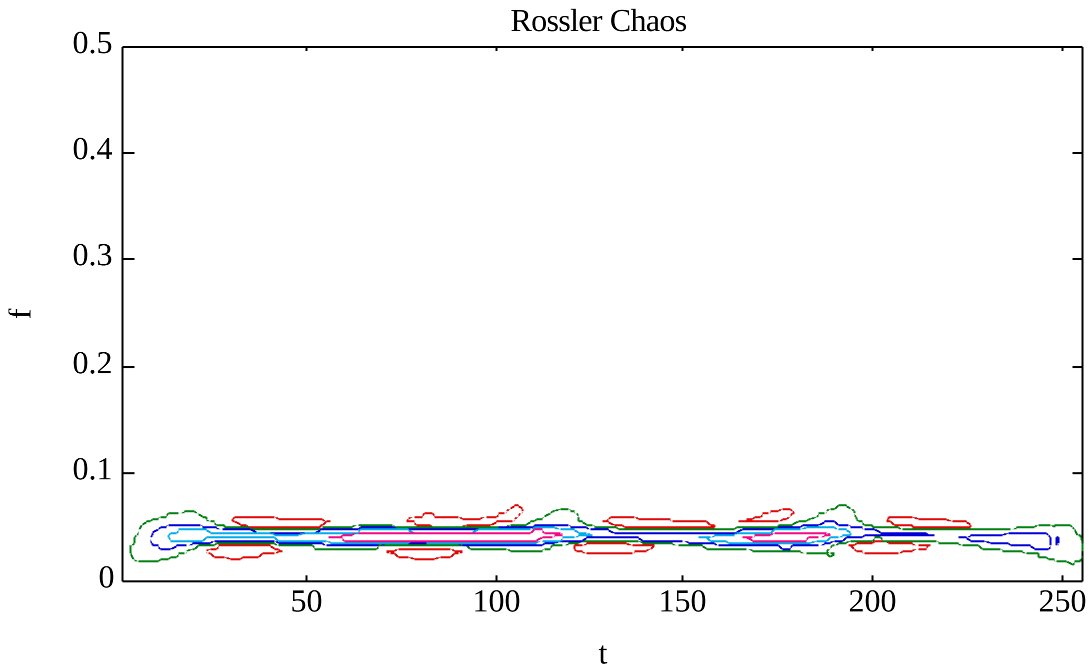
In testing deterministic chaos, the power spectrum plays an important role in studying color chaos in laboratory experiments [Swinney and Gollub 1978]. In testing chaotic signals, thousands to hundreds of thousands' of data points are required by the power spectrum. The noise level should be kept between 2 to 5 percent. The WGQ spectrogram has much stronger power to distinguish deterministic cycles from stochastic noise [Chen and Qian 1993a, Chen 1993c].

The strong power of time-frequency analysis can be understood from the energy distribution of signals. White noise is evenly distributed in time-frequency space while deterministic cycles are highly localized. The noise level in the power spectrum is an integration of the energy distribution in the time domain; therefore, the time-frequency distribution has a much higher signal/noise ratio than that of the power spectrum. For example, the autoregressive (AR) model can produce artificial cycles in the power spectrum. It cannot generate a stable frequency line in time-frequency representation [Chen and Qian 1993].

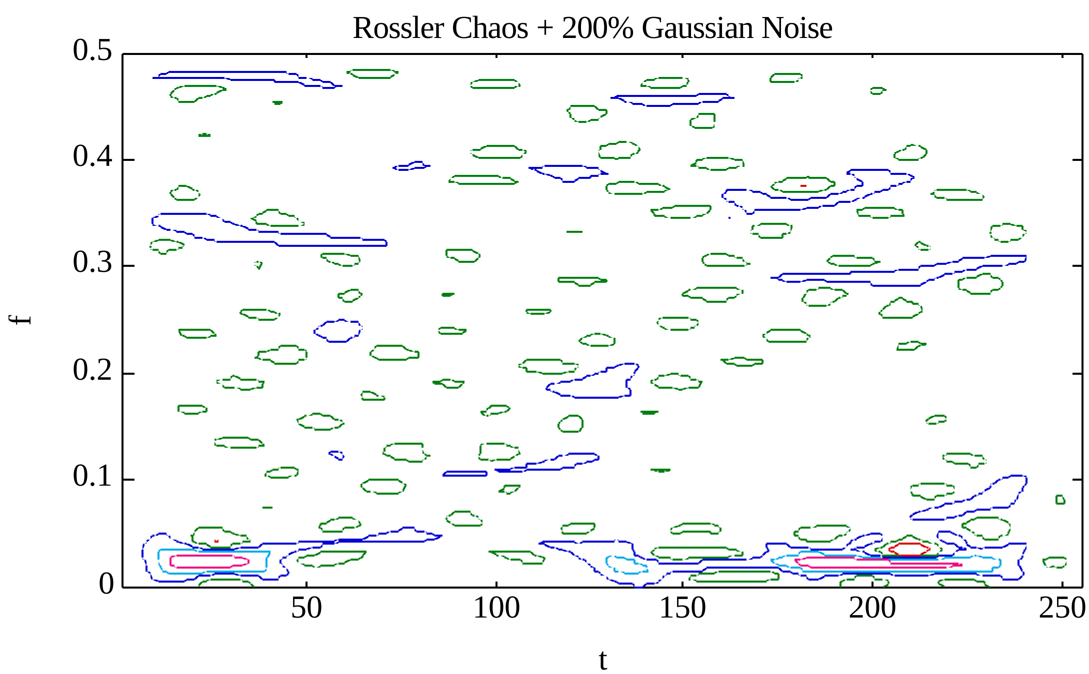
Econometric analysis assumes that all economic variables are random variables. From the view of signal processing and pattern recognition, testing mixed signals of cycles and noise is a more realistic task. Our investigation will focus in this direction. Typical WGQ representations of noise and chaos are shown in Fig. 1.



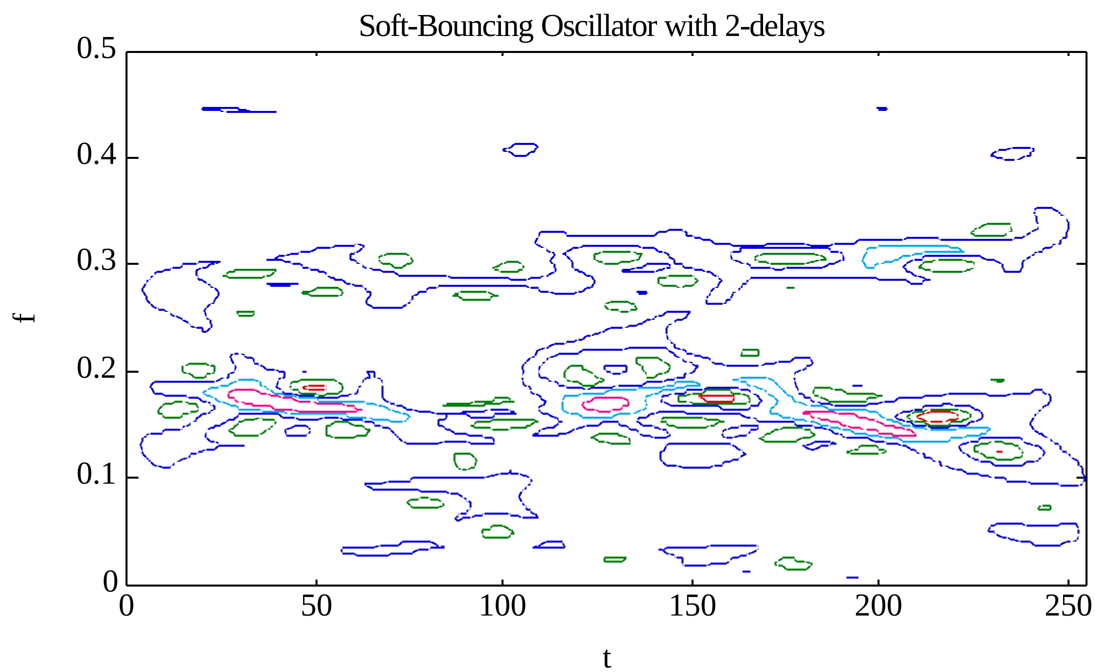
(1a). Gaussian white noise. No stable frequency line exists.



(1b) Rössler strange attractor is generated by three-dimensional differential equations: dX / dt =–Y-Z, dY / dt= X+0.2Y, dZ / dt= 0.2-5.7Z+XZ (Rössler 1976). The time unit is adjusted to appear a business cycle frequency.



(1c) Color noise modeled by Rössler chaos plus 200% noise (measured by the standard deviation).



(1d) Color chaos generated by the soft-bouncing oscillator with two time delays (Wen 1994). $dX/dt = 100\ X(t-0.183)\ \exp[-200\ X(t-0.183)^2] - 6\ X(t-0.183) - Y(t-0.048)$, $dY/dt = X(t)$.

Fig. 1. The WGQ spectrogram of noise and chaos. N=256.

Under WGQ representation, deterministic signals are characterized by a localized horizontal zone in the time-frequency space, while noise signals are featured by drop-like images distributed in wide time-frequency space. The patterns of color noise and color chaos are not easy to distinguish under numerical analysis, especially in the case of high-dimensional chaos [Wen, Chen, and Turner 1994, Wen 1993, 1994].

III. Trend-Cycle Decomposition and Noise-Cycles Identification

The detrending problem in business cycle studies is closely related to the observation reference in economic theory. Many controversial issues in macroeconomic studies, such as the over-smoothness of consumption, the excess volatility of stock prices, and the debate of chaos versus noise in economic aggregates, are closely related to competing detrending methods [Hall 1978, Shiller 1989, Brock and Sayers 1988, Barnett and Chen 1988, Chen 1988, 1993a].

Two most popular detrending methods are log-linear detrending and differencing detrending. Their theoretical frameworks are called trend-stationary (TS) and difference-stationary (DS) time series. The HP filter is a generalization of the log-linear detrending [Hodrick and Prescott 1981].

Econometric studies of detrending filters are based on a key assumption that economic time series can be characterized by linear stochastic processes [Nelson and Plosser 1982]. The main analytical tools are correlation analysis and frequency analysis of stationary process [King and Rebelo 1993]. The whole picture will be quite different, if testing signals are not generated by white noise, but by color noise. In testing the performance of FD and HP filters, we use simulated time series of color noise and a wide range of empirical data, including sixteen economic aggregates.

(3.1) Correlation Cancellor (FD) and Trend Smoothing Filter (HP)

The differencing procedure can be considered as a linear filter f(L) or F(L), with L as the lag operator.

$$Y(t) = X(t)-X(t-1) = \Delta X(t) = f(L)\, X(t) = (1-L)\, X(t) \qquad (3.1)$$

$$X(t) = F(L)\, Y(t) = (1-L)^{-1} Y(t) \qquad (3.2)$$

The differencing is a non-invertible filter with marginal stability. Its main function in econometric modeling is a correlation cancellor. Actually, the differencing is not a whitening device but a "violeting" one, since it dampens low frequency components but amplifies high frequency components. Differencing generates an erratic time series when the time unit is not small as compared to serial correlations. The discontinuity caused by differencing can be described by a step function whose Fourier transform is a delta function [Papoulis 1977]; therefore, differencing may introduce a zero-frequency (dc) component. This often happens with trendy time series.

An alternative way is to find a smooth trend by fitting log-linear or polynomial functions. A difficulty is the choice of period boundaries in trend removing. This problem can be alleviated by the Hodrick-Prescott (HP) "trend smoothing" algorithm [Hodrick and Prescott 1980].

The HP filter is a linear transformation of the original time series { X(i) } into a smooth time series { G(i) } by minimizing the following objective function

$$Min \sum [X(t)-G(t)]^2 + \lambda \sum \{[G(t+1)-G(t)]-[G(t)-G(t-1)]\}^2 \qquad (3.3)$$

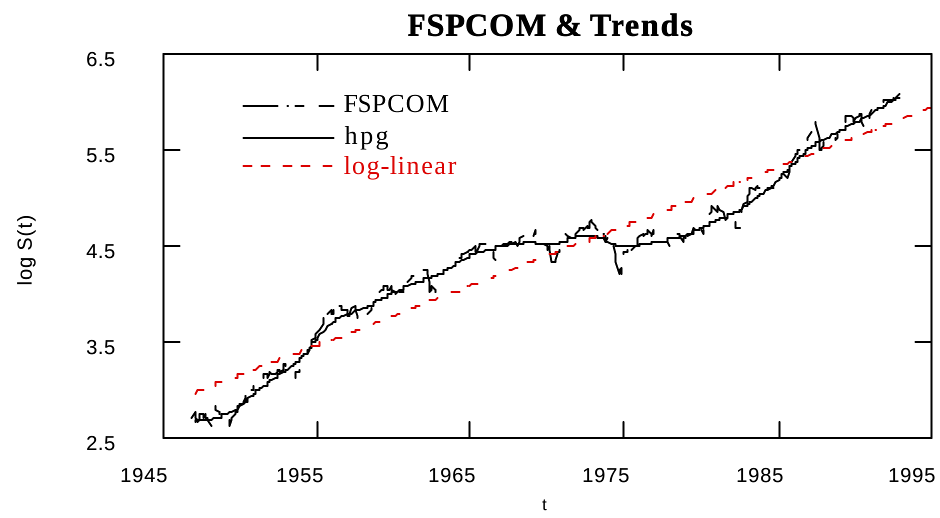
Deviations from { S(i) } are considered as the cyclic component:

$$C(i) = X(i) - G(i) \qquad (3.4)$$

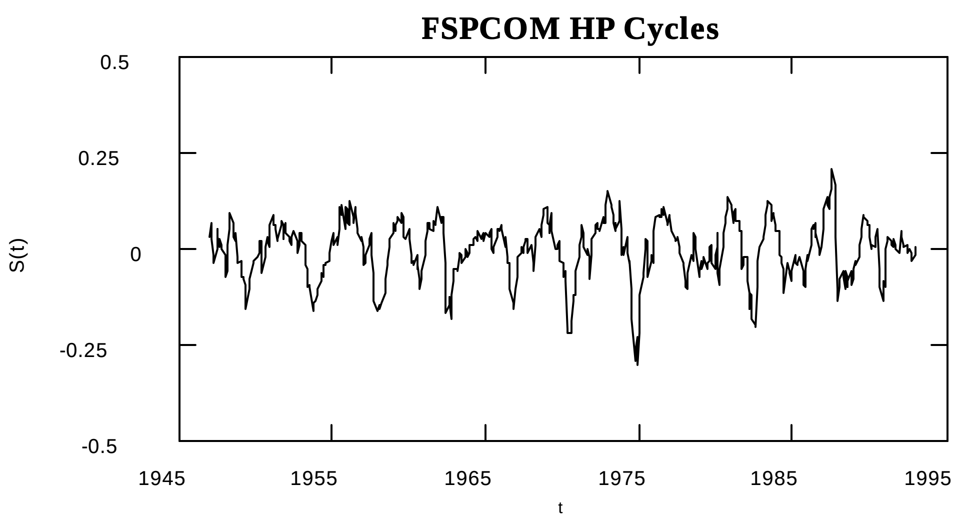
Empirical time series can be decomposed into "smooth" growth series { G(i) } and cyclic series { C(i) }. The characteristic period of HP short cycles depends on the penalty parameter of λ. λ is chosen in such a way that the variance of the growth component is much less than that of the cyclic term [Hodrick and Prescott 1980]. In practice, the recommended value of λ is 400 for annual data, 1600 for quarterly data, and 14400 for monthly data.

The penalty term in Eqn. (3.3) is the second difference in the growth series. When λ goes to infinity, the growth trend is a linear function. For logarithmic data, log-linear detrending corresponds to the limiting case in HP decomposition. HP growth trends are less rigid than the log-linear function and HP cycles are less erratic than differencing. Certainly, HP growth trends provide little information about growth cycles and long waves. A more generalized algorithm of multi-level symmetric decomposor will be further developed to analyze multiple frequencies in business cycles [Chen and Qian 1993].

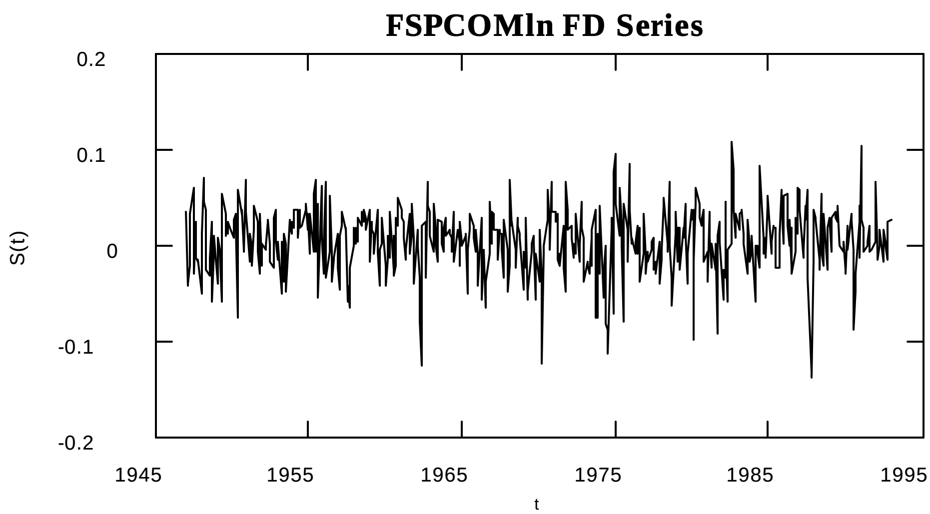
A typical example of economic time series is showed in Fig. 2. The erratic feature of FD series and the wavelike feature of TS cycles are visible from their autocorrelations.



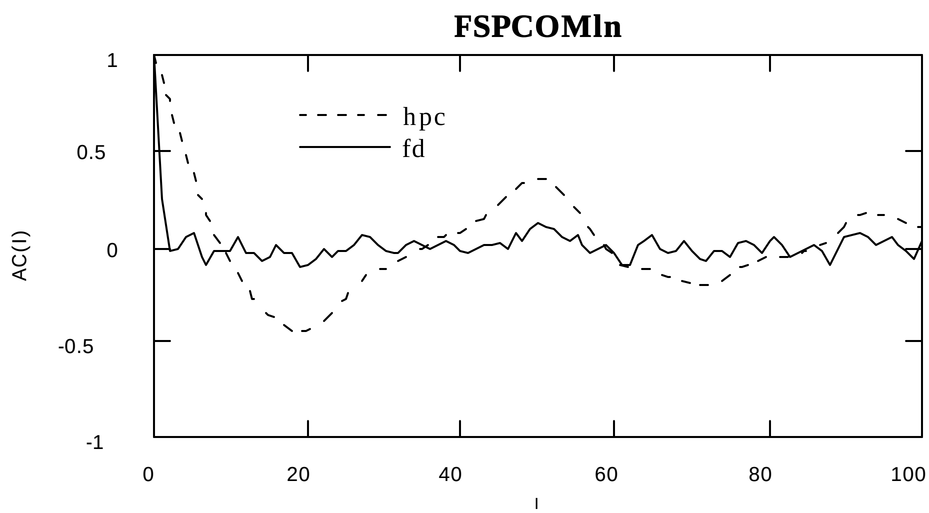
(2a) Log-linear trend and HP growth trend.



(2b) TS (trend-stationary) series from HP (Hodrick-Prescott) filter.



(2c) DS (difference-stationary) series form FD (first difference) filter.



(2d) Autocorrelations of HP and FD cycles.

Fig. 2. The logarithmic FSPCOM (the S&P 500 stock price index) monthly data (1947-92). N=552.

(3.2) Correlation Analysis of Noise and Cycles

Correlation analysis is capable of revealing the existence of deterministic cycles when we examine cyclic movements in serial correlations. We may define the decorrelation time T measured by the lag length of the first zero in autocorrelations [Chen 1988, 1993a]. Usually, the time lags in correlation analysis are integers. Here, the fractal length of decorrelation time is calculated from linear interpolation in the framework of continuous time.

The decorrelation period P_{dc} can be defined as following:

$$P_{dc} = 4T\ \Delta t \tag{3.5}$$

where Δt is the time unit of the time series.

For deterministic cycles, P_{dc} is close to the characteristic period P_c measured by the peak in the power spectrum. For random signals, T has no implication of cyclic movement.

We can see that the FD filtered time series have a shorter T or smaller P_{dc} as compared to HP filtered series [Table I].

Table I. Correlation and Variance Analysis of Filtered Time Series

Series	Δt	Period	N	λ	σ_{hp}	T_{hp}	σ_{fd}	T_{fd}
GDPQ*	Q	1947-92	184	1600	0.0180	4.83	0.0102	3.51
LBOUTU*	Q	1947-92	184	1600	0.0104	4.22	0.0081	3.40
GCQ*	Q	1947-92	184	1600	0.0117	4.93	0.0077	3.72
GCDQ*	Q	1947-92	184	1600	0.0547	4.95	0.0416	2.58
GPIQ*	Q	1947-92	184	1600	0.0822	3.73	0.0535	2.71
FSPCOM*	M	1947-92	552	14400	0.0750	8.93	0.0340	1.95
FSDXP	M	1947-92	552	14400	0.3420	8.41	0.1670	1.84
FYGT10	M	1953-92	480	14400	0.6305	9.80	0.3198	1.73
FM1*	M	1959-92	408	14400	0.0116	11.03	0.0049	20.84
FM2*	M	1959-92	408	14400	0.0099	11.37	0.0034	24.87
GMYFM2*	M	1947-92	552	14400	0.0154	10.3	0.0073	4.99
LHUR	M	1948-92	540	14400	0.6398	9.38	0.2340	8.94
PZRNEW*	M	1947-92	552	14400	0.0103	11.99	0.0040	88.82
FYFF	M	1955-92	456	14400	1.2898	10.64	0.6377	1.98
FYCP90	M	1971-92	264	14400	1.3860	10.5	2.4630	1.79
EXRJAN	M	1959-92	408	14400	11.270	9.70	4.7500	5.68

where the time unit (Δt) is Q (quarter) for quarterly data and M (month) for monthly data; N, number of observations; λ, the HP parameter; σ, the standard deviation; T, the decorrelation time (in Δt). Subscripts of hp and fd are for HP and FD series respectively.

Among the empirical time series, GDPQ is the real gross domestic products in 1987 US dollars, LBOUTU is the non-farm output per hour; GCQ is the total consumption; GCDQ is the durable consumption; GPIQ is the domestic investment; FSPCOM is the S&P 500 composite monthly index; FSDXP is the S&P common stock composite dividend yield; FYGT10 is the 10 year Treasure Notes; FM1 is the Federal Reserve monetary supply M1 index; FM2 is the Federal Reserve monetary supply M2 index; GMYFM2 is the velocity of money; LHUR is the unemployment rate; PZRNEW is the consumer price index for all items; FYFF is the rate of Federal Funds; FYCP90 is the three month commercial paper rate and EXRJAN is the exchange rate of Japanese Yen vs. U.S. dollar. All quantity data marked by the * symbol are in logarithm. The source of these data is the Citibank Database.

We should point out that the very long T_{fd} for FM1 and FM2 differenced data is caused by residual trends in first differenced data. These are good examples that multiple differencing may be needed to remove trends.

(3.3) Characterizing the randomness and instability in the frequency domain

In a time series analysis, the degree of whiteness is often examined by its autocorrelations in the time domain. We will introduce some useful indicators of randomness and instability in the frequency domain.

Given a time series S(t), t = 1, 2, . . . , T, we can calculate its power spectrum R_i, i = 1, 2, . . . , M. We define γ as the degree of randomness of a time series in terms of the discrete-time information entropy in the frequency domain:

$$g = \frac{\sum p_i (\log_2 p_i)}{y} \qquad (3.7)$$

$$p_i = \frac{R_i}{\sum_{i=1}^{M} R_i} \qquad (3.8)$$

$$\psi = \log_2 M \tag{3.9}$$

Here, R_i is the power intensity of frequency i calculated from the power spectrum; p_i, the probability of frequency i and M, the number of states in the frequency domain. ψ is the normalization factor which is equal to the maximum entropy of white noise, whose $p_i = M^{-1}$. In ideal cases, γ is zero for periodic motion and 1 for white noise. The degree of randomness of color chaos or color noise will fall in between. In numerical tests, r is less than 0.3 for periodic cycles, and larger than 0.9 for the Gaussian noise depending on the size of data.

From the time-frequency distribution F(f,t), we can identify the peak frequency distribution f(t) and calculate useful statistics to characterize peak frequency f(t). For changing frequency of nonstationary time series, the characteristic frequency f_c is a function of time. The peak frequency f(t) can be determined at each time intersection in a time-frequency representation. The characteristic frequency f_c can be measured by the mean value of the peak frequency. Its frequency instability can be defined by the standard deviation of the peak frequency.

We define ς as the degree of frequency instability measured by the percentage of white noise frequency bandwidth:

$$\varsigma = \mathrm{std}(f(t)) / W \tag{3.10}$$

Here W=0.5 for the full band window in spectral analysis. For stable periodic cycles, ς is near zero. For random process, ς is close to one. The frequency instability can be considered as a measure of internal randomness caused by frequency evolution over time. For example, a harmonic oscillator with wandering frequency may appear as random signals in Fourier spectrum, even though its deterministic nature can be seen from time-frequency analysis

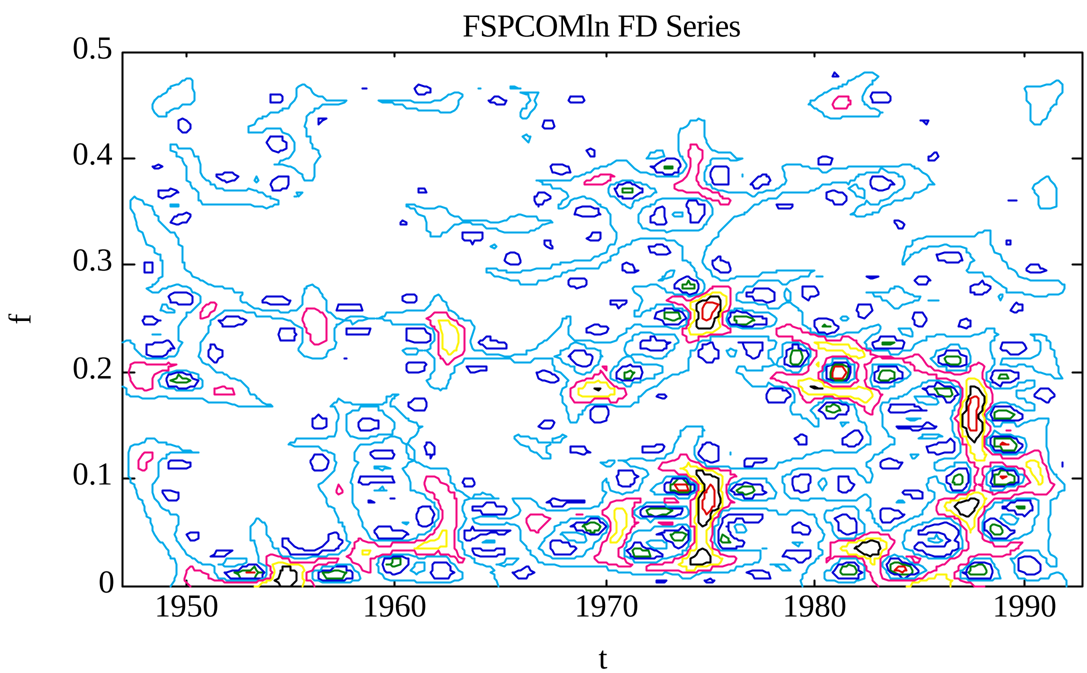
Similarly, we may also define the frequency variability v as the percentage ratio of the standard deviation to the mean frequency.

$$\nu = \mathrm{std}(f) / f_{mean} * 100\% \tag{3.11}$$

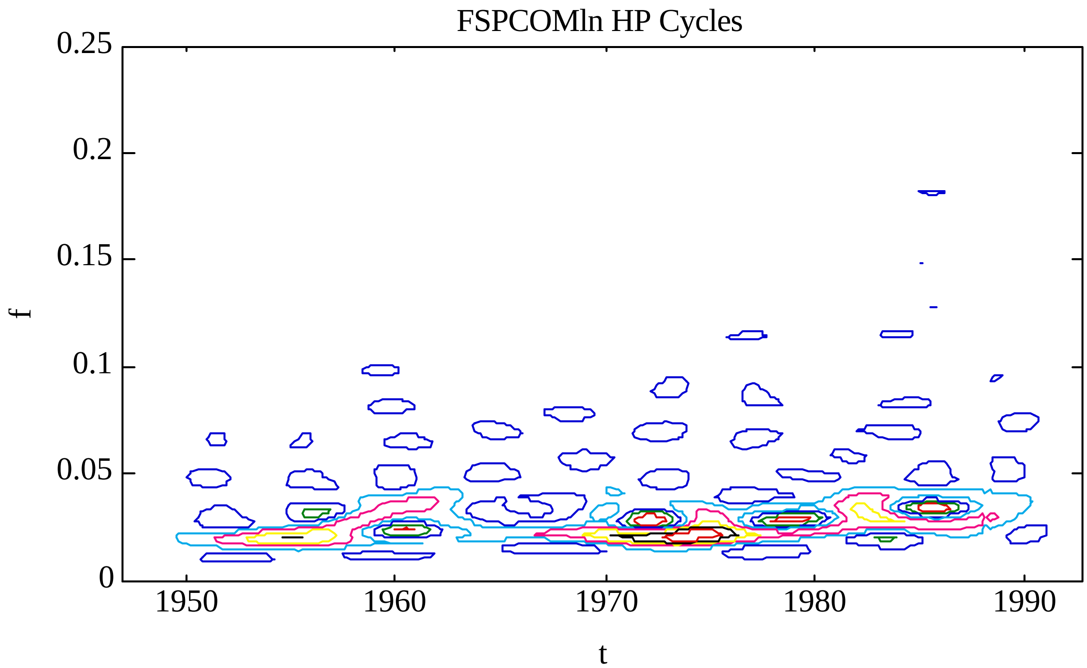
We will use the above quantitative measures in studies of filter performance for mixed signals of noise and cycles.

(3.4) Color Residuals and Time Unit Consistency

We use the FSPCOM time series to demonstrate the performance of FD and HP filter in studies of business cycles. WGQ spectrograms are given in Fig. 3. The residual statistics under time-frequency representation are given in Table II. The residual statistics in the frequency domain under different time units are given in Table III. We can see that these residuals are far from white noise in spectral analysis.



(3a). First differences.



(3b). HP cycles (λ=14400).

Fig. 3. The WGQ Spectrogram of the logarithmic FSPCOM N=552.

Table II. Time-Frequency Analysis of FSPCOM Filtered Cycles

Filter	f_{mean}	std(f)	ς	ν	P_{mean}	P_{min}	P_{max}
HP (λ=14400)	0.0265	0.0057	0.0114	22	3.1	0.70	4.73
FD	0.0893	0.0886	0.1772	99	0.9	0.25	inf

Here, the time unit of periods is a year. The sampling time interval Δt is 1/12 of a year; std(f), the standard deviation of peak frequency over time; $f_c = f_{mean}$; $P_c = P_{mean} = \Delta t / f_{mean}$; P_{min} and P_{max}, the range of peak period over time; ς, the degree of frequency instability and ν, the frequency variability (%).

Table III. Spectral and Correlation Analysis with Changing Time Unit

Name	Δt	Filter	γ	P_c(yrs)	$T(\Delta t)$	P_{dc}(yrs)	std(t)
FSPCOM	M	FD	0.8895	inf	1.9	0.6	0.0338
	Q_f	FD	0.8831	inf	1.4	1.4	0.0707
	Q_v	FD	0.8384	inf	2.0	2.0	0.0578
	A_f	FD	0.7075	inf	0.1	0.4	0.1136
	A_v	FD	0.6475	inf	1.5	6.1	0.0895
	M	HP (λ=14400)	0.5501	3.6	8.9	3.6	0.0752
	Q_f	HP (λ=1600)	0.7366	4.0	3.5	3.5	0.0892
	Q_v	HP (λ=1600)	0.6659	3.8	3.5	3.5	0.0834
	A_f	HP (λ=400)	0.7982	3.8	1.8	7.1	0.1161
	A_v	HP (λ=400)	0.6592	5.1	1.7	6.7	0.0863

Here, γ is the frequency information entropy indicating the degree of randomness; P_c, the characteristic period in power spectrum; T, the decorrelation time; P_{dc}, the decorrelation period and std(t), the standard deviation in time domain.
Two methods of constructing a time series in a larger time unit are used. Q_f (A_f) series are constructed by picking up the figure of the final month (quarter) in the season (year). Q_v (A_v) series are constructed by averaging value in the season (year).

We can see that the FD filter does not provide a consistent picture of a detrended series. The frequency information entropy indicates that the FD residuals are not white. The band width of the FD residuals is less than 20% of white noise. There is a strong component at near zero-frequency caused by the discontinuous nature of differencing time series. The FD filter fails to produce a consistent picture under the changing time unit. Changing the time unit will change the length of decorrelation time and the magnitude of variance. The FD filter plays a destructive role in testing the cyclic signals. The time-frequency representation shows that noisy signals of high frequencies are strongly amplified, while the deterministic cycles in the range of business cycles are hard to recognize from the FD filtered series. The negative effects of the FD filter are not visible for pure deterministic or pure stochastic signals, but are quite severe for noisy data with growth trends.

In contrast, the HP filter provides a consistent picture of persistent cycles from an economic time series when the sampling rate is large enough to detect business cycles (quarterly or monthly, but not annual data). The characteristic periods for economic aggregates are highly stable, since they are slightly changing over time. The frequency variability of HP cycles is as low as less than three percent. The characteristic period P_c from spectral analysis and decorrelation period P_{dc} from correlation analysis are remarkably close. This is strong evidence of deterministic cycles. The magnitude of the characteristic period P_c is essentially invariant under the changing time unit. Time unit consistency paves the way for refined measurement and generalized theory.

Previous claims of unit roots in aggregate data are produced from fitting annual or quarterly data to low-order ARMA (autoregressive and moving average) models [Nelson and Plosser 1982, Campbell and Mankiw 1987]. There is no evidence of the unit root process since FD-filtered quarterly and monthly data are far from white under spectral representation [Chen 1993c].

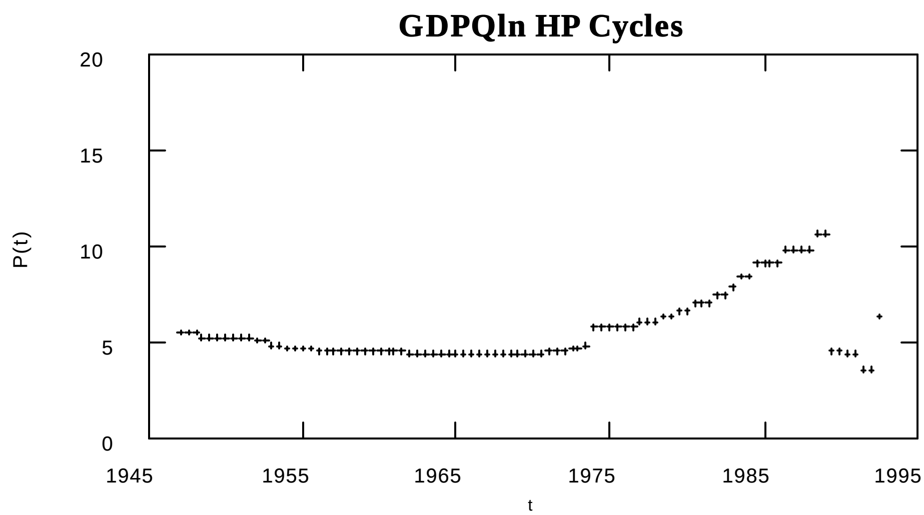
IV. Frequency Patterns and Dynamical Changes in Business Cycles

The HP filtered economic time series show clear evidence of persistence cycles in the time scale of business cycles defined by NBER documentation [Zarnowitz 1992]. We will further examine their frequency patterns and structural changes by time-frequency analysis.

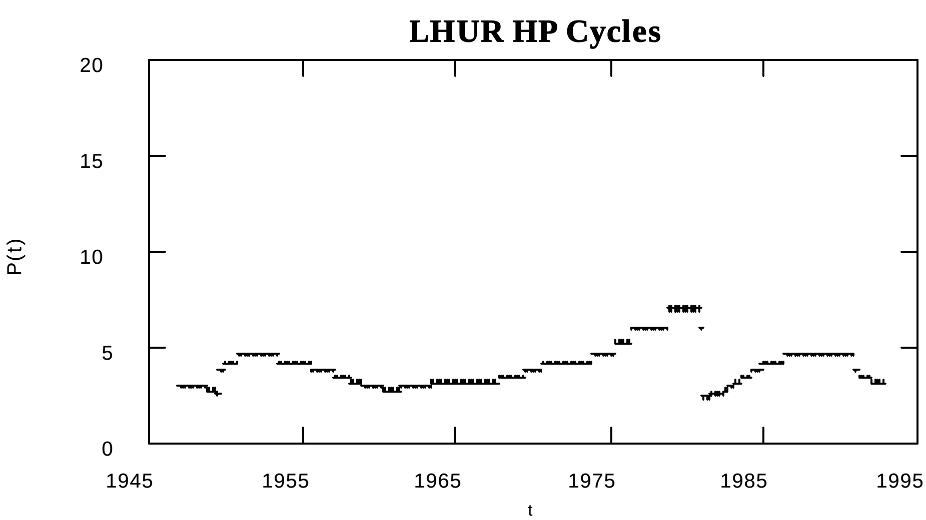
In econometric modeling and business cycle theory, variance-correlation analysis is the main tool in characterizing volatility and propagation mechanism [Kydland and Prescott 1990]. Structural changes are described by parameter changes in parametric models [Perron 1989, Friedman and Kuttner 1992]. The time-frequency representation provides a new tool in observing dynamical patterns in business cycles.

(4.1) Frequency Stability and Structural Flexibility

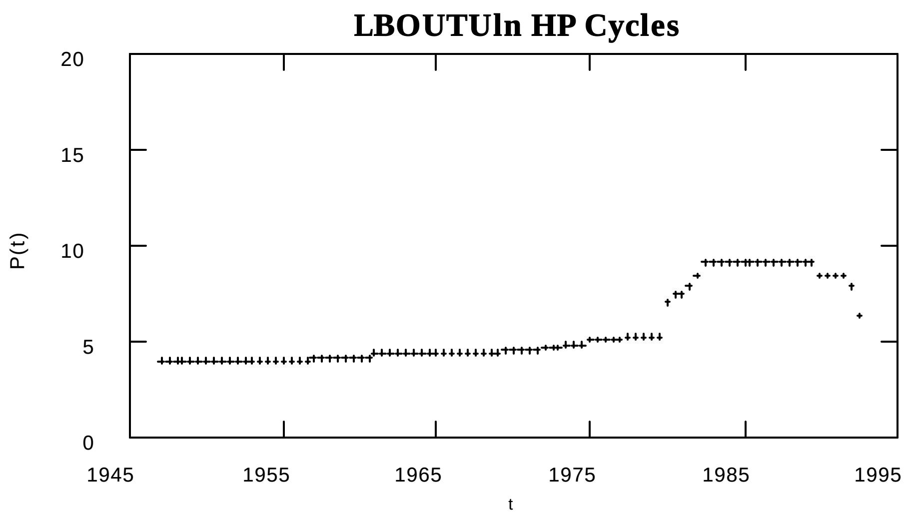
We tested a wide range of aggregate data. Most of them have distinct color, or characteristic frequency. The empirical results of sixteen economic aggregates from time-frequency analysis are given in Table IV. The period evolution for general indicators, such as GDPQ, LHUR, and LBOUTU are shown in Fig. 4.



(4a). GDPQ (the real gross domestic products) HP cycles. N=184.



(4b). LHUR (the unemployment rate) HP cycles. N=540.



(4c) LBOUTU (the labor productivity) HP cycles. N=184.

Fig. 4. Period Evolution of General Indicators.

We find that the only frequency break of GDPQ HP cycles was caused by the first oil price shock in 1973. This observation provides complementary support to trend-shifting argument based on the parametric test [Perron 1989].

However, the oil price shock only had a minor impact on most macroeconomic indicators. For LHUR HP cycles, the first frequency shift occurred within the Korea War, the second and more dramatic change appeared in early 1980's. For LBOUTU, a significant change happened in the early 1980's. Most economic indicators show more complex patterns of frequency evolution in history.

Table IV. Frequency Stability and Variability of HP Short Cycles

Series	Δt	Period	N	P_{dc}	P_c(yrs)	ν (%)
GDPQ	Q	1947-92	184	4.8	5.4	23
LBOUTU	Q	1947-92	184	5.1	4.2	26
GCQ	Q	1947-92	184	4.4	4.9	47
GCDQ	Q	1947-92	184	4.4	5.0	49
GPIQ	Q	1947-92	184	4.4	3.7	34
FSPCOM	M	1947-92	552	3.1	3.0	22
FSDXP	M	1947-92	552	2.9	2.8	37
FYGT10	M	1953-92	480	3.1	3.3	37
FM1	M	1959-92	408	3.7	3.6	46
FM2	M	1959-92	408	3.9	3.3	46
GMYFM2	M	1947-92	552	3.8	3.4	32
LHUR	M	1948-92	540	3.9	3.1	23
PZRNEW	M	1947-92	552	4.0	4.0	27
FYFF	M	1955-92	456	3.6	3.5	51
FYCP90	M	1971-92	264	3.1	3.5	73
EXRJAN	M	1959-92	408	3.0	3.2	57

Here, P_{dc} is the decorrelation period from correlation analysis; P_c, the characteristic period from time-frequency analysis and ν and frequency variability [see Fig. 5-6].

Examining the frequency stability of HP cycles under a time-frequency representation, the characteristic frequency of LBOUTU is most stable, while those of FYFF and FYCP90 are most variable. The other aggregates are in between. There are several interesting observations to business cycle studies.

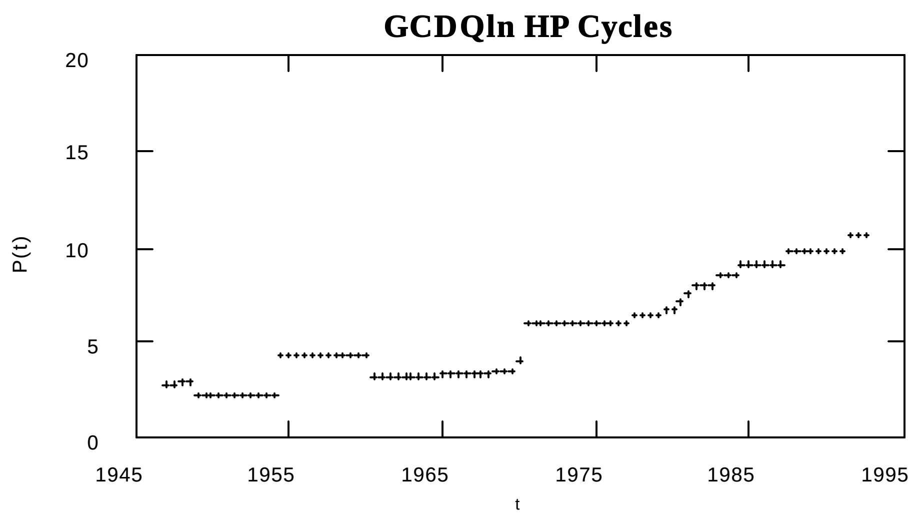
First, the frequency stability of economic indicators is remarkable. The variability of frequency is less than 80 percent. The band width is only about 1 to 5 percent of white noise. Specifically, monetary movements cannot be oversimplified as external shocks because money indicators also have stable characteristic frequencies. The monetary velocity is more stable than FM2 and the long-term interest rate is more stable than the short-term interest rate.

Second, these characteristic frequencies have the similar range of magnitudes, but a distinctive pattern; therefore they are nonlinear oscillators in nature, because a linear combination cannot change the characteristic frequencies.

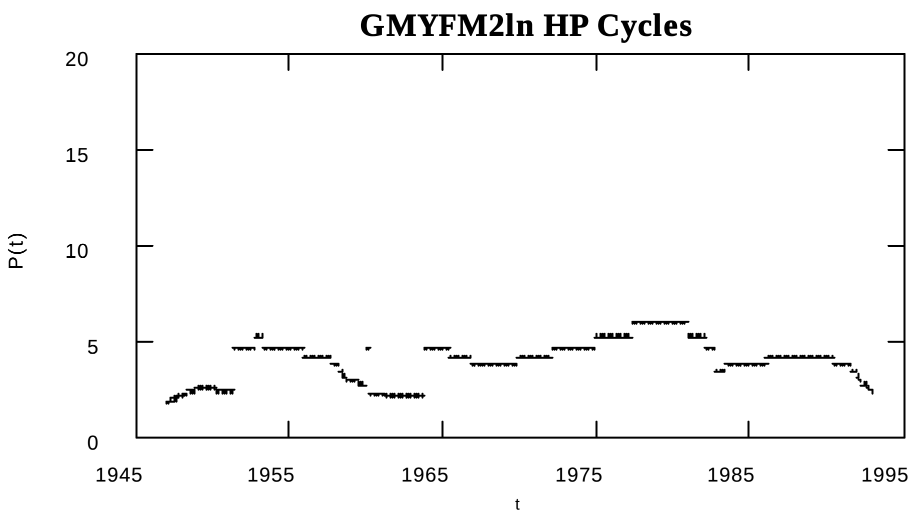
Third, the stability and flexibility of the characteristic frequency under constant shocks cannot be explained by the Frisch-type linear oscillators [Frisch 1933]. High-dimensional nonlinear oscillators are needed to describe persistent cycles observed from economic data.

(4.2) Frequency Evolution and Pattern Classification

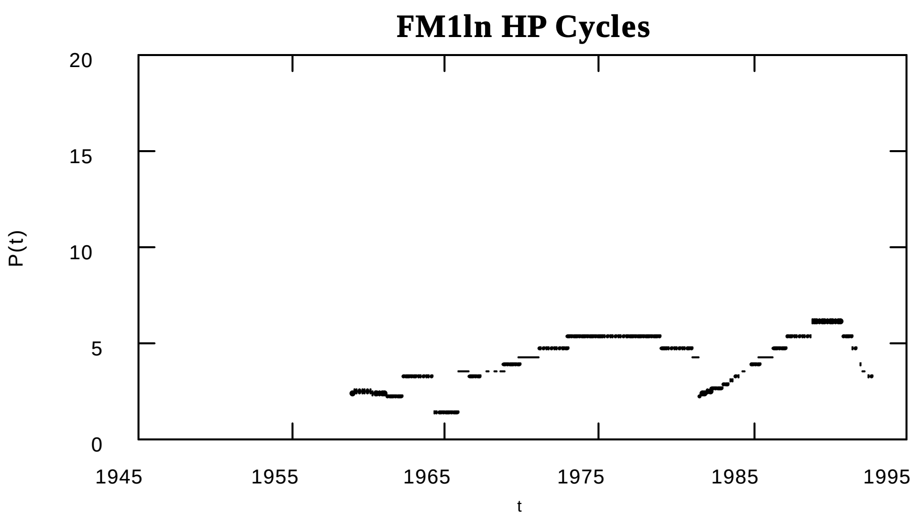
Econometric modeling are used to treat economic aggregates as homogenous random variables. Under time-frequency representation, we find hard and soft cycles from their distinct patterns of frequency evolution [Fig. 5].



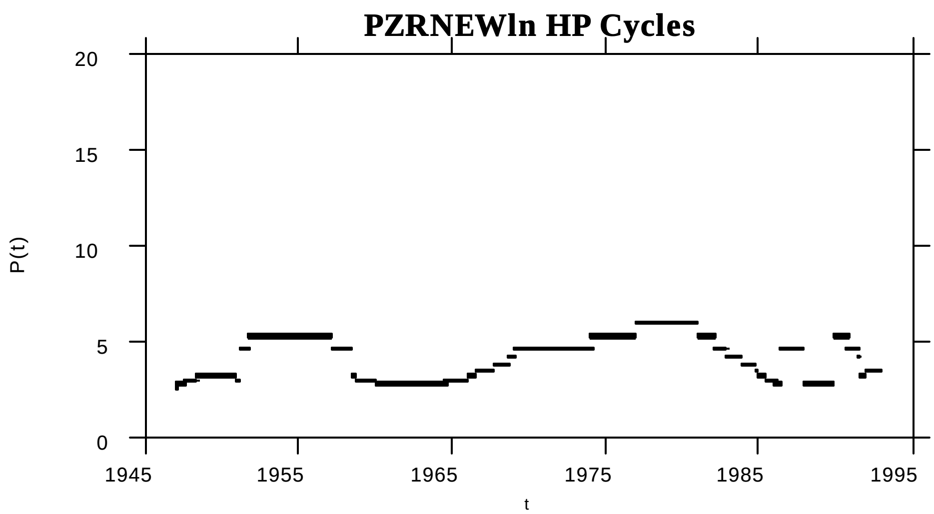
(5a) GCDQ (the real durable consumption) HP Cycles. N=184.



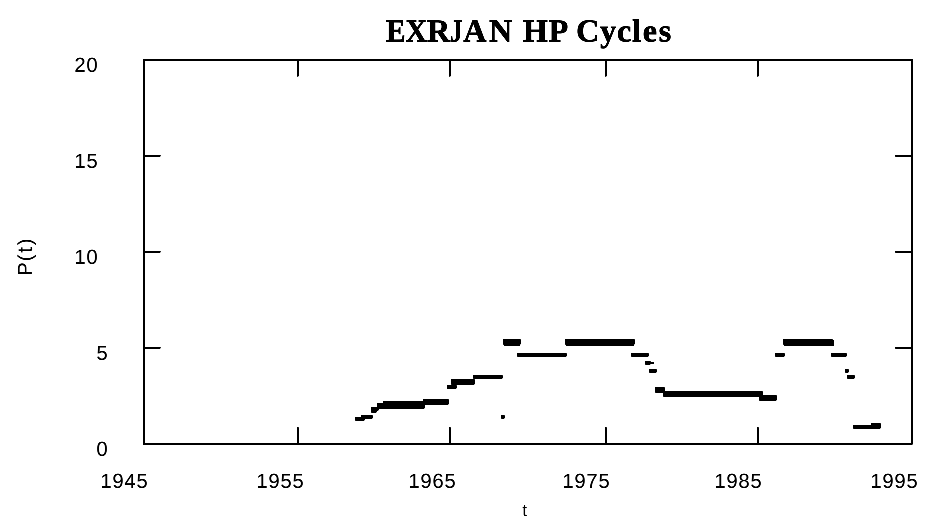
(5b) GMYFM2 (the monetary velocity) HP Cycles. N=552.



(5c) FM1 (the money supply M1 index) HP cycles. N=408.



(5d) PZRNEW (the consumer price index) HP cycles. N=552.



(5e) EXRJAN (the Japanese / US exchange rate) HP Cycles. N=408.

Fig. 5. Period Evolution of Hard & Soft Cycles

Consumption, investment, and productivity are examples of hard cycles. They have piece-wise flat regimes, a reflection of stability and rigidity in frequency domain. Hard cycles are more stable against small changes but vulnerable under dramatic shocks. Hard cycles behave like an autonomous subsystem such as the circulatory system and digestive system in humans. It is conceivable that consumption, investment, and technology have their own dynamics.

Stock market indexes, monetary velocity, money supply, the consumer price index, interest rate, and exchange rate are examples of soft cycles. Soft cycles tend to move together since they have similar patterns in time-frequency space. A new kind of frequency co-movements reveals a close interaction between stock market, money market, and economic performance.

We can further identify subgroups of economic indicators based on their pattern of similarity in frequency evolution. For example, both GDPQ and LBOUTU are insensitive to most historical events. GCQ, GDPQ, and GPIQ have similar rigidity and stability. Two stock market indicators, FSPCOM and FSDXP, and the long-term interest rate FYGT10 have almost the same pattern, even though their frequencies are not the same. The pairs of FYFF and FYCP90 also move together during frequency shifting. These observations provide useful information about the interacting mechanism and propagation dynamics. Examination of the time-frequency pattern will be a valuable guidance for business cycle modeling.

For example, consumption and investment have closer interactions than income and price.

Monetary movements have less an impact on stock market and the long-term interest rate than on the short-term interest rate.

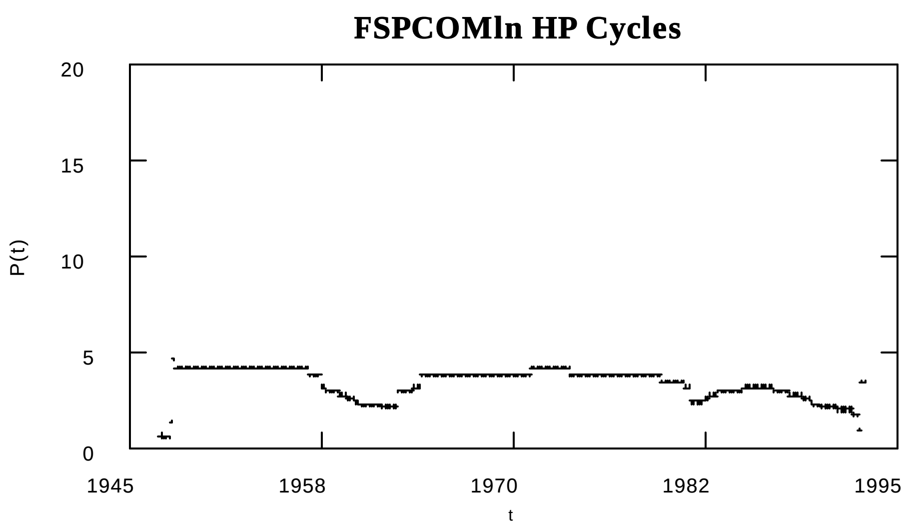
The role of money is not neutral in business cycles. The frequency pattern of monetary indicators are similar to that of the consumer price index and the unemployment rate but more variable than real income, investment, and consumption. Unlike seasonal changes in weather, government and monetary authority are integrated players in economic dynamics. Monetary movements have complex structures.

The real GNP serves as an anchor in real business cycle modeling [Kydland and Prescott 1990]. However, real GNP is not a sensitive indicator for structural changes. The monthly data of the unemployment rate can be a better barometer of business cycles and structural changes.

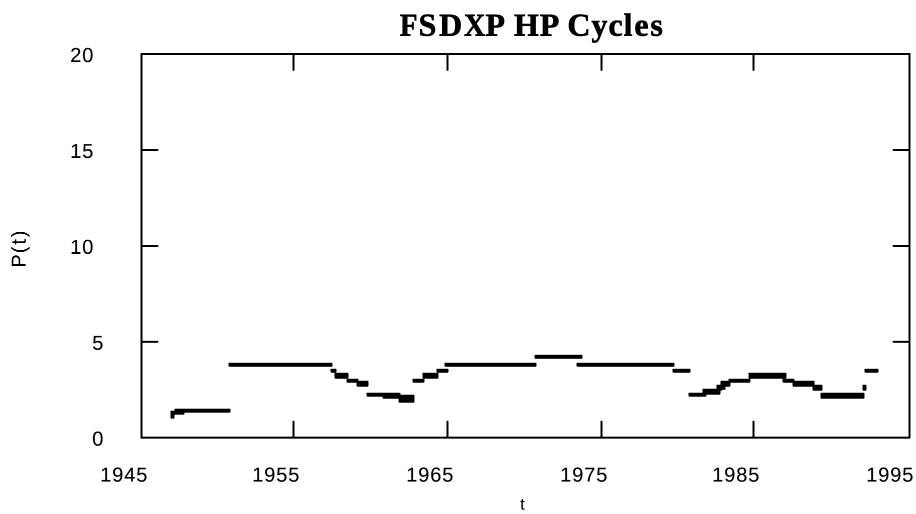
(4.3) Breaking Points and Propagation Mechanism

The breaking points in frequency evolution provide explicit information about the propagation mechanism. We can observe propagation speed and delay process by reviewing historical events.

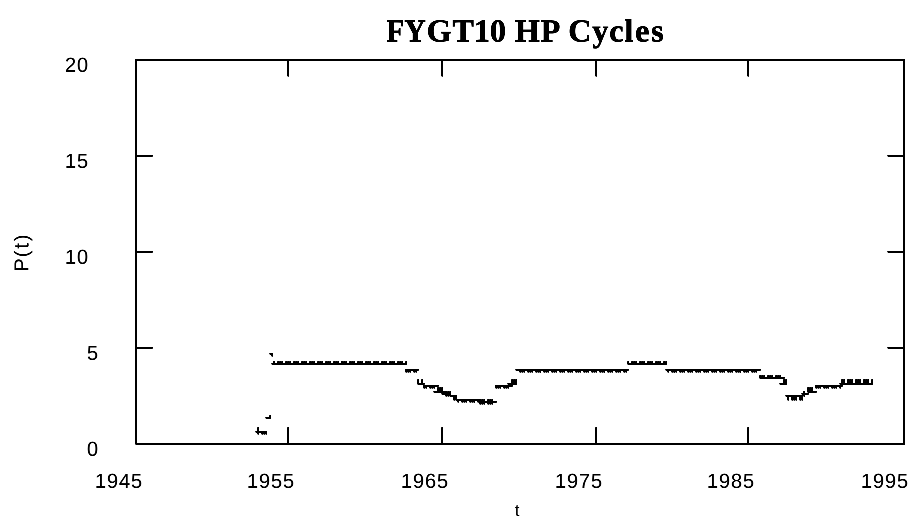
In econometric exercises, the issue of persistent shocks is not clear under regression analysis [Christiano and Eichenbaum 1990]. Impacts of historical shocks vary greatly under time-frequency representation. For example, two pairs of economic aggregates, stock market indexes of FSPCOM and FSDXP, the Federal fund rate FYFF and the short term interest rate FYCP90, behave like synchronous cycles. In contrast, the frequency pattern of the long-term interest rate FYGT10 almost duplicates the pattern of stock market indicators. But the frequency hysteresis lasted about six years during the Vietnam War and oil price shocks [Fig. 6].



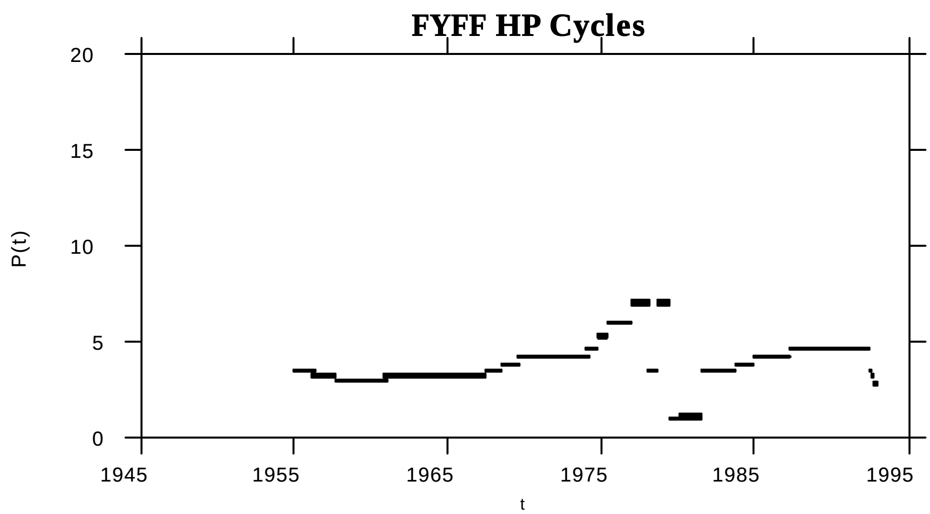
(6a). FSPCOM (the S&P 500 stock index) HP Cycles. N=552.



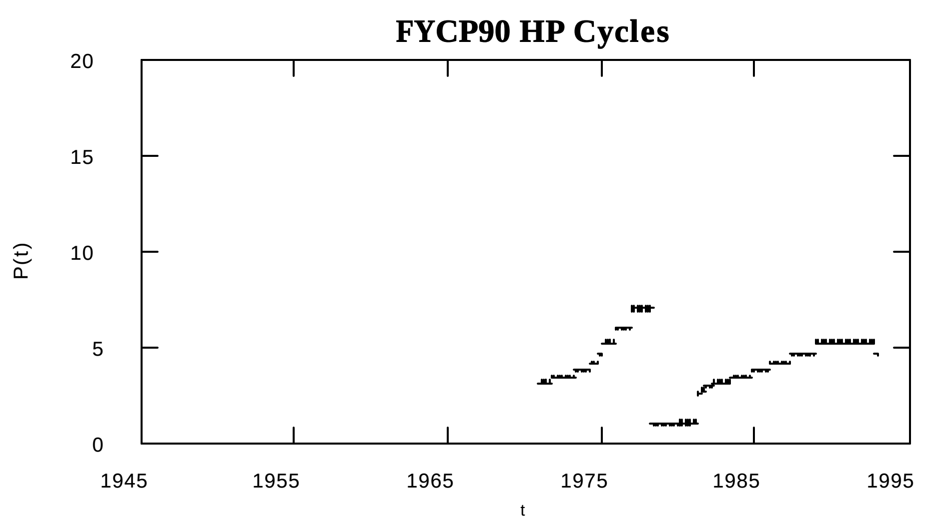
(6b) FSDXP (the S&P stock dividend yield) HP Cycles. N=552.



(6c). FYGT10 (the 10 year treasure notes rate) HP Cycles. N=480.



(6d). FYFF (the rate of Federal Funds) HP Cycles. N=456.



(6e). FYCP90 (the 3-month commercial paper rate) HP Cycles. N=264.

Fig. 6. Synchronous Cycles and Frequency Hysteresis

Exceptional variability in Federal Fund rates and short-term interest rates are visible in Fig. 6. It is known that the pattern of monetary movements had changed greatly in 1980's [Friedman and Kuttner 1992]. The puzzling issue of "missing money" and other anomalies can be explained away by adding more variables, such as long-term and short-term interest rates, in the error-correction model [Baba, Hendry, and Starr 1992]. This approach is skeptical under time-frequency analysis since variables with different frequency responses cannot be easily put together in linear models.

(4.4) Oil Shocks, Stock Market Crash, and the Vietnam War

The extraordinary resilience of market economies can be revealed from the insignificance of the first oil price shock in October 1973 and the stock market crash in October 1987. Both events generated only slight changes of characteristic frequencies for most economic aggregates.

We may have a closer look at frequency evolution in observing historical events [Fig. 6a, b]. Before the oil price shock in October 1973, the characteristic period of stock market indicators was stabilized at the level of 4.26 years since 1971. After the oil price shock, the characteristic period of HP cycles changed to 3.86 years. Obviously, the oil price shock was the external cause of frequency change in the stock market.

This may not be the case for the stock market crash in October 1987. There was a long swing of frequency during the 1981-1990 period. For FSPCOM and FSDXP HP cycles, their characteristic period of 3.28 years lasted for two years (1985-86). Then, their characteristic period slightly changed to 3.05 years for FSPCOM (January - December 1987) and FSDXP (January - October 1987), and 2.84 years thereafter. The stock market crash happened at the end of the 10-month "frequency shift." There was a 2-month delay for FSPCOM after the stock market crash. This suggests that the stock market crash is the end of an internal bubble instead of external shocks.

The interesting thing is that the political economy of wars and arm races left a much stronger fingerprint in the time-frequency representation. The most significant changes of the frequency pattern happened in three periods: US-Soviet arm and space races during 1958-1962, the escalation of the Vietnam War during 1965-1972, and the so-called Reagan revolution in 1980's. Not only fiscal and monetary policy, but also industrial policy, tax policy, and military program, may have notable impacts on structural changes of US economy.

V. Theoretical Implications of Persistent Cycles and Economic Instabilities

There is no question that external noise and measurement errors widely exist in economic data. The question is whether some regularities are observable from an empirical time series. The answer is yes.

Like a telescope in astronomy or a microscope in biology, time-frequency analysis opens a new window of observing evolving economies. The most enlightening result in business cycle studies is the discovery of persistent cycles, i.e., self-generating cycles from economic aggregates. These cycles are nonlinear in nature with remarkable resilience and flexibility like living beings. This discovery provides a new perspective to business cycles. Traditionally, the economic order is characterized by negative feedback and equilibrium (steady) states. The new role of persistent cycles challenges the linear framework of economic dynamics. We need to re-examine the implications of complexity and instability in business cycles.

(5.1) Characteristic Frequencies and Endogenous Cycles

The existence of characteristic frequencies in economic movements has profound implications in business cycle theory.

Economic movements, like organisms, have their distinct time rhythms. Different economic factors move with different speeds and different frequencies. In this sense, economic aggregates have their "personalities" and they are not all alike in frequency patterns. The pattern recognition in economic dynamics will pave the way for economic diagnostics and policy valuation.

Changing patterns of characteristic frequencies of business cycles reveal internal sources of economic shocks, such as military expenditures and tax policies. Economic interactions are highly correlated and are an essential nature of collective phenomena. The impact of monetary shocks and technology shocks can be better understood if we know their own dynamics. There is no absolute dividing line between internal and external shocks. Nonlinear interaction, rather than linear causality, provides a better picture for understanding the historical experience of economic evolution.

We should point out that the popular name of "chaos" is somewhat misleading because of its negative image of irregularity and disorder. That is why we suggest the term of "complex cycles" for deterministic chaos. We also prefer the name of "color chaos" to white noise. Unlike controlled experiments in natural science, complex chaotic cycles may not be "verified" in economic dynamics, but can be observed through empirical patterns, such as time-frequency representation. Like calculus for classical mechanics, Riemann geometry for gravitation theory, new mathematical tools, such as nonlinear dynamics and nonstationary time series analysis, are critical to the advancement of economic dynamics.

(5.2) Time Scale and Observation Reference in Measurement and Theory

Econometricians often argue that the measurement method of economic indicators (such as annually or daily data) demands the discrete-time algorithm [Granger and Teräsvirta 1993]. These technical arguments ignore fundamental issues of time scale in any dynamical theory [Chen 1993a, b].

In the history of empirical science, many theoretical controversies can be settled by refining measurements. Whether economic laws of motion (if they exist) are invariant under changes of time units is a fundamental issue in economic dynamics. For example, if econometric tests of long-run economic relationship, such as unit roots or linear causality, succeeds for annual or quarterly data but fails for monthly data, the validity of underlying economic theory would be questionable [Lütkepohl 1991].

The most visible pattern of economic movements is the recurrent feature of business cycles in the time scale of several years. Regression analysis is more comfortable with annual data than monthly data, since serial correlations can be easily explained by stochastic models with few lags. However, annual data is helpless for spectral analysis. Measurement precision does matter in empirical economic studies. If dynamic patterns change with the time scale, such as the patterns of the stock market movements during a trading day may differ from the patterns during a business cycle, we should change the dynamic model and not just the time unit.

The degree of mathematical complexity is associated with computational reliability. Conventional discrete-time ARMA models have poor resolution because of the extremely low computational degree of freedom. A better resolution of the WGQ spectrogram comes from new representation reconstruction in terms of a two-dimensional Gaussian lattice instead of a one-dimensional polynomial fitting.

The differencing operator serves a poor reference base in business cycle studies. From the view of resource constraints, the DS framework implies unlimited resources in economic dynamics, if level information is not relevant to economic dynamics. Most economic variables, including the government budget, credit limits, wealth, capital stock, savings, inventory, consumption, and production, are measured by levels. Rich patterns in HP cycles indicate that both flow and level variables matter in economic dynamics.

We recommend the HP filter as a better device in trend-cycle decomposition because the HP filter produces consistent measurement and historical patterns through time-frequency analysis. We may consider HP long cycles as a long-run evolving equilibrium and model HP short cycles by strange attractors.

(5.3) Dynamical Instability and Information Ambiguity

In the history of science, some thought experiments once dramatically shaped theoretical thinking in fundamental issues. Notable examples are: Maxwell's demon in thermodynamics, the uncertainty principle in quantum mechanics, and the Friedman paradox on the nonexistence of destabilizing patterns in speculative dynamics.

Friedman asserts that no predictable pattern can exist in the market beyond a short time horizon because rational arbitrageurs (Friedman's spirits) will rapidly wipe out any destabilizing traders from the market [Friedman 1969]. This is the essence of the efficient market hypothesis and the main argument against the possible existence of market regularity. Actually, Friedman's spirits behave much like Maxwell's demon in equilibrium thermodynamics, although their purposes are just the opposite [Chen 1993b, Brillouin 1962].

In addition to information costs and financial constraints [Grossman and Stiglitz 1980, De Long, Shleifer, Summers, and Waldmann 1990], there are more serious barriers for arbitrageurs' action.

First, the observational reference for economic equilibrium and market fundamentals are simply not well-defined operationally. Friedman's argument may be valid for an island economy without growth and nonlinear interactions among residents, but not valid for an open economy with growth and collective actions.

Second, the problem of information ambiguity is more fundamental than information scarcity from the point of view of time-frequency analysis. The implications of the information flow can only be understood in terms of a historical context, such as the case of linguistic analysis. It is impossible to judge economic trends from uncorrelated shocks. Investment hysteresis in the range of two to four years can be understood by the value of waiting under uncertainty [Dixit 1992]. The time delay in information analysis and decision making is a main source of overshooting and inertia.

Third, dynamical instability and bounded rationality set fundamental limits to economic forecasting. Nonlinearity, nonstationarity, and the uncertainty principle in information analysis, all contribute to complexity and indeterminacy in economic forecasting [Prigogine 1993, Chen 1993a, b]. Friedman's argument implies that irrational speculators are sure losers. This would be true only for simple dynamical systems when market movements could be perfectly predictable. The extreme cases of complete unpredictability of random walks and perfect predictability of harmonic cycles are unrealistic features of linear dynamics. The modest behavior of nonlinear oscillators fills in the gap between the two extremes.

In short, rational arbitrageurs on average cannot eliminate cyclic patterns, even in the long run. We can forecast general economic trends including their mean period and variance, but we cannot predict time path and turning points even when we know of some pattern of economic dynamics. There is no sure winners or losers on the speculative market because of the complex nature of business cycles.

VII. Brief Conclusion: Evolving Economy and Complex Dynamics

Now, we have a better understanding of why business cycles have been well documented by the NBER approach, but are hard to characterize by statistical analysis based on a stationary process [Zarnowitz 1992]. The existence of growing economic trends and structural changes needs new analytical tools for nonlinear and nonstationary process. Time-frequency representation and the HP trend-cycle decomposition pave the way to study persistent cycles from empirical economic data.

The characteristic frequencies of economic variables provide rich information about internal dynamics and structural changes. Our integrated approach in empirical analysis and theoretical framework reveals the important role of time scale, observation reference and pattern recognition in business cycle studies.

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