Origin of Division of Labor and Stochastic Mechanism of Differentiation*

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Abstract: Nonlinear models are introduced to describe the nonequilibrium dynamics of social evolution. The difference between Western and Oriental culture, and their roles in the origin in the division of labor, are described by a behavioral model in information diffusion and learning competition. It shows a trade-off between stability and diversity. The stochastic mechanism of social differentiation and the empirical evidence for this is discussed in a stochastic model of multi-staged development. It shows that the break-down of the Gaussian distribution during a transition. Finally, an ideal model of social bifurcation is given.

Key Words: Information diffusion, learning, competition, behavior, division of labor, bifurcation

1. Introduction

It is recognized that social evolution takes place in the context of a nonequilibrium world. But mathematical models in social sciences have in the past been dominated by the equilibrium paradigm. Today, a new methodology and new paradigm is appearing in nonequilibrium physics and chemistry [1]. Its impact on social sciences is increasing [2].

In this short paper, we will discuss two simple models of nonequilibrium process in social phenomena. In section 2, we first introduce the behavioral factor in the learning process and the cultural trait in a competition model. We find a trade-off between security and development or stability and variability. This sheds some light on the differences between the Occidental and Oriental culture and the origin of division of labor in history. In section 3,we discuss the stochastic mechanism for the breakdown of Gaussian distribution. We first discuss a stochastic model of multi-staged development. Its initial and final state is a Gaussian distribution, but a multi-humped distribution appears during the transition stage. We also discuss the evidence for

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this stochastic mechanism in social differentiation. Finally, we give an ideal case of social bifurcation.

2. A behavioral model in learning competition: the origin of division of labor

Why there is a striking contrast between the Occidental and Oriental culture? Why did science and capitalism emerge in Western Europe but not in China, India, Islamic and other civilizations? Modern economics seems to lack working economic models about the origin of division of labor. In theoretical biology, the outcome of competing species essentially depends on resources and environment.[3] There is no link between changes in environment and adaptation from animal or human behavior.

It is realized that the culture factors play an important role in the origin of capitalism and sciences[4][5]. For example, M.Kikuchi is greatly aware of differences in the degree of "individualism" existing in the Eastern and Western nations. He suggests an one-dimensional model of degree of individualism, such as an axis ranging from highly individualistic European countries and the US at one extreme to the society of honey bees at the other[6]. We will discuss this point from the view of learning process. We introduce a simple behavioral or cultural trait to study the mechanism of division of labor. First, we will study the modified information diffusion model for one species, then we will consider learning competition, a model with two species.

2.1. An information diffusion model

Let us consider an information diffusion process without a central information source. We assume :

$$\frac{\partial n}{\partial t} = k\, n\, (N - n) - d\, n\, (1 - \alpha \frac{n}{N}) \qquad (2.1.1)$$

Here N is population size, n is the number of knowers, N-n is the number of learners, k is the growth rate. The last term is the removal rate or forgetting rate. Its form differs from the conventional constant loss rate [7]. We may consider d as the measure of learning ability or degree of difficulty in learning a new technology.

Here we introduce a new factor &a, the degree of sensitivity. If $\alpha > 0$, it is a measure of strangeness aversion. When few people accept the new information, the removal rate is large. When most people accept it, the forgetting rate decreases. This is the characteristic of conservatism. On the contrary, If $\alpha < 0$, this term is a measure of adventure loving. The absolute value of &a is less or equal to unity. Different &a represent different behavior or cultures, such as, social or solitary animals, conservative or progressive cultures. We easily find the equilibrium solution to (2.1.1)

$$n^* = N\left(1-\frac{d}{kN}\right) / \left(1-\frac{d\alpha}{kN}\right) \qquad (2.1.2)$$

We see that

$$n^*_{\alpha<0} < n^*_{\alpha=0} < n^*_{\alpha>0} \qquad (2.1.3)$$

If we have a fluctuating environment, we may consider following stochastic equation:

$$\frac{\partial x}{\partial t} = k\,x\,(N - x) - d\,x\left(1 - \alpha\frac{x}{N}\right) + \sigma\,k\,x\,\xi(t) \qquad (2.1.4)$$

Here x is a random variable, $\xi(t)$ is white noise, σ is the variance of white noise.

It is easy to find the extrema of the stationary probability density of the Fokker-Planck equation obtained from (2.1.4) [8]. They are :

$$x_m = N\left(1 - \frac{d}{kN} - \frac{k\alpha^2}{2N}\right) / \left(1-\frac{d\alpha}{kN}\right) \qquad \text{when } \sigma < \sigma_c \qquad (2.1.5)$$

$$x_m = 0 \qquad \text{when } \sigma > \sigma_c \qquad (2.1.6)$$

$$\text{where} \quad \sigma_c^2 = \frac{2}{k}\left(N-\frac{d}{k}\right) \qquad (2.1.7)$$

We may compare two species: one is conservative in learning ($\alpha_1 > 0$) and another is progressive($\alpha_2 < 0$). For conservative species, their steady size n_1 is larger. So, in order to maintain the same population size, they need smaller resource. And for progressive species n_2, they need larger subsistence space. For stability against fluctuating environment, the conservative one is more stable than the progressive. It is especially true if there exists some survival threshold population[9]. But when new information comes, the conservative species has less potential to absorb new technology than progressive, if the learning ability is limited for every individual.

2.2. A learning competition model

Now we consider the learning competition model for two species with different cultures.

$$\frac{\partial n_1}{\partial t} = k_1\, n_1(N_1 - n_1 - n_2) - d_1\, n_1\left(1 - \alpha_1\frac{n_1}{N}\right)$$

$$\frac{\partial n_2}{\partial t} = k_2\, n_2(N_2 - n_2 - n_1) - d_2\, n_2\left(1 - \alpha_2\frac{n_2}{N}\right) \qquad (2.2.1)$$

Here $n_{1,}$ n_2 are knowers in species one and two respectively. We also assume $k_1 = k_2 = k$ and $N_1 = N_2 = N$ for simplicity.

We may rewrite the equation as follows:

$$\frac{\partial n_1}{\partial t} = s_1\, n_1(M_1 - n_1 - \beta_{12}\, n_2)$$

$$\frac{\partial n_2}{\partial t} = s_2\, n_2\, (M_2 - n_2 - \beta_{21}\, n_1) \qquad (2.2.2)$$

Here $M_{i,}$ $s_{i,}$ β_{ij} are the effective carrying capacity of resources, effective growth rate and effective competition coefficient respectively. And i, j is 1 or 2.

We have

$$M_1 = (N - \frac{d_i}{k}) / (1 - \frac{\alpha_i d_i}{kN})$$

$$s_i = k\, (1 - \frac{\alpha_i d_i}{kN})$$

$$\beta_{ij} = 1 / (1 - \frac{\alpha_j d_j}{kN}) \qquad (2.2.3)$$

Note: here we have asymmetric overlap coefficients β_{ij}, and β_{ij} may larger than 1 when α_j is less than zero. This result differs from the conventional competition equations [10].

From (2.2.2), we find the condition for coexistent species. That is

$$\beta_i\, M_i < M_i < M_i / \beta_j$$

or :

$$(1 - \frac{\alpha_1 d_1}{kN})\, (1 - \frac{\alpha_2 d_2}{kN}) > 1 \qquad (2.2.4)$$

We immediately find that two conservative species can not coexist. When they compete for the same resource, such as arable land, the only result is one replaces the other. It is the story repeatedly occurring in a traditional peasant society. Division of labor can not emerge in a conservative culture.

If two species have equal learning ability ($d_1 = d_2$), then two progressive species coexist, but the conservative species will replace the progressive one. So, the only strategy for progressive species in competition is to improve their learning ability (to have smaller d). If we consider capitalism as a culture of adventure loving, then we may reach the similar conclusion as economist Joseph A.Schumpeter that innovation is vital for capitalism in the competition between East and West[11]. Once innovations stop ,capitalism will lose the game in the competition for existing resources.

If $d_1 \neq d_2$ and $\alpha_1 \neq \alpha_2$, there are variety of possibility for competing species, therefore, we have a diversified world.

We may also study the stability against a fluctuating environment in terms of coupled Fokker-Planck equations. It is already know that the stability of a two coexistent species system is less stable than single one[12]. Or we can say that a monolithic society is more stable than pluralistic one, although pluralistic society enjoys more social wealth than the monolithic has. There is a trade-off between security and development or between stability and diversity [13]. Division of labor has its benefit and cost.

Based on our model, we may discuss the evolutionary tree of social history. Clearly, it is a two way motion towards simplicity or complexity, depending on the environment and the structure of the system. We might also speculate why capitalism emerged in the West and not in the East.

3. The stochastic mechanism for the breakdown of Gaussian distribution in social differentiation

The Gaussian distribution is widely used in sciences. Because it is simple in application, we only need two parameters, the mean value and variance, to characterize a stochastic system. The peak evolution of Gaussian distribution generally follows the solution of corresponding deterministic equation[14]. The limitation of the Gaussian distribution lies in The fact that it is a simplified model of near a equilibrium system. When fluctuations are very large, the mean value becomes a meaningless concept. In order to understand the mechanism of multi-humped distribution, it is necessary to go beyond the limits of the equilibrium dynamics.

We first give a numerical example of multi-humped distribution. Then we discuss an ideal case of bifurcation in social phenomena.

3.1. A stochastic model of multi-staged growth

An early studies of long tail or second hump of distribution was done in the epidemic model[15]. Recently, the work in explosion model shows the properties of fluctuations during transition stage[16]. Here we consider a general model of pure birth process, the master equation is given as following:

$$\frac{\partial P(n,t)}{\partial t} = b(n-1)\,P(n-1,t) - b(n)\,P(n,t) \qquad (3.1.1)$$

Note $P(n,0) = \delta(1)$ and $b(0) = b(N) = 0$. Here n can be regarded as population or state number. The transition probability from n to n+1 in time t to $t+\Delta t$ is b(n). If the birth rate b(n) is not a smooth function but a piecewise one, which is common seen in multi-staged development process, we will easily generate a multi-humped distribution function during transition stages. A numerical example of time evolution in distribution function is shown in Fig.1

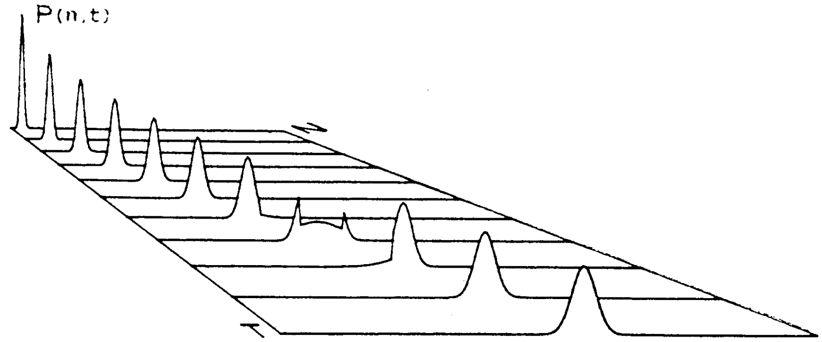

Fig. 1. Time evolution of probability distribution in pure birth process. We choose N = 1000, b = 1.0 for 350 < x <450, otherwise, b = 0.3. Here the scale of time depends on the constant of birth rate.

This simple model reveals the stochastic origin in social differentiation during multi-staged development. Since any social system has finite size N, the law of large numbers can not rule out the existence of non-Gaussian distribution in social phenomena.

The stochastic mechanism of differentiation is very important to understand social development. Suppose we do a survey of a group of children, examining their physical and psychological development. We will certainly find deviations from average behavior. Some children may advance far ahead, and some fall far behind. Social scientists might attribute these differences to some genetic, socio-economic or cultural factors, because they assume the normal development will follow the Gaussian distribution. Failing to identify these hidden factors, one might treat it as a kind of error in measurement, simply ignoring these distribution tails. In contrast, we suggest that even when we have a true homogeneous multi-humped distributions under conditions far from equilibrium. For example, the learning process cannot be a solely deterministic. Chance plays an important role in human development.

However, to prove the stochastic mechanism is not an easy task. In experiments, we need a large sample in a cross-age survey or a trace of a homogeneous group for a long time. In theory, the definition and measurement of development degree may be questionable for many models in social sciences. Without a sufficient number of states, an averaging procedure could easily hide a multi-humped distribution. Homogeneity is also a controversial concept, because it is difficult to exclude any hidden varia bles.

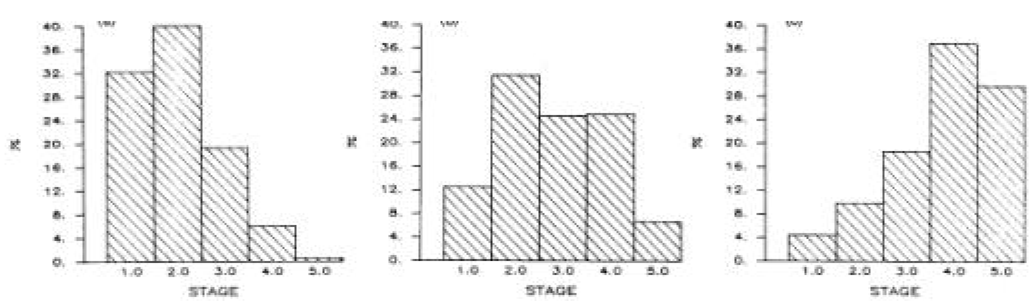

Fig. 2. U.S. survey of sexual maturation of white boys. The abscissa is the genital stage. N is the sample size.

In spite of this, some social scientist data still encourages us to search the evidence of multi-humped distributions. One possible example is shown in Fig. 2. A survey in the U.S. examines the sexual maturation of boys and girls. A broaden distribution in white girls of age 12 and a slightly dip in distribution for white boys of age13 are seen [17][18]. A similar flat distribution are found by Belgian data too[19]. At least, it is an evidence of large fluctuations during this transition period. The breakdown of Gaussian distribution is a clear sign of non-equilibrium state. Certainly, further study in multi-staged process is worthwhile.

3.2. The bifurcation at the last meal

One may raise the question of the existence of bifurcations in social phenomena [20]. Because social scientists cannot do experiments, it seems impossible to observe bifurcation in social history.

To answer this question, let us consider an ideal case. Suppose a castle is encircled by overwhelming hostile forces. The castle is hard to attack both from land and sea. The best choice for the people in the castle is to stay inside as long as their food store lasts. But at the last meal, chaos spreads among the people. They face three choices. The old strategy to stay in the castle becomes unwise, since it only leads to die of hunger. And the survival chance to escape by land or sea seems the same. Some people prefer one way and some prefer the other. Obviously, there is a bifurcation. The bifurcation parameter is the food store and the state space is discrete. This story is not unusual in history and repeated often. Deterministic bifurcation theory is the average of this sort of the events.

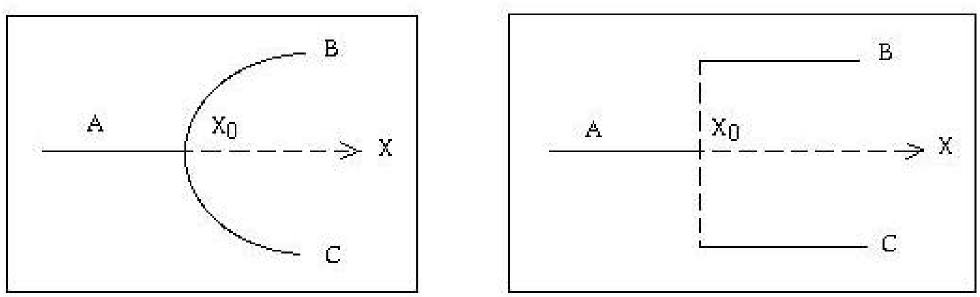

Fig.3. Bifurcation diagram of defense strategy in an island castle. The stock of food is the bifurcation parameter. X_0 is the bifurcation point.

Although the above case is an ideal one, we can easily see that the concepts of bifurcation and multi-humped distribution are useful to describe social phenomena.

4. Concluding Remarks

From the above simple models, we see how the stochastic mechanism plays an important role in social evolution. Developments in nonequilibrium physics and chemistry have not only an impact on technical problems of social sciences, but it leads to dramatic modifications of our concepts or philosophy of social theory. The equilibrium paradigm of a static world will be replaced by the nonequilibrium paradigm of an evolutionary world.

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