

Transition Probability, Dynamic Regimes, and the Critical Point of Financial Crisis

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Abstract

An empirical and theoretical analysis of financial crises is conducted based on statistical mechanics in non-equilibrium physics. The transition probability provides a new tool for diagnosing a changing market. Both calm and turbulent markets can be described by the birth-death process for price movements driven by identical agents. The transition probability in a time window can be estimated from stock market indexes. Positive and negative feedback trading behaviors can be revealed by the upper and lower curves in transition probability. Three dynamic regimes are discovered from two time periods including linear, quasi-linear, and nonlinear patterns. There is a clear link between liberalization policy and market nonlinearity. Numerical estimation of a market turning point is close to the historical event of the U.S. 2008 financial crisis.

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1.Introduction

How to diagnose the nature of business cycles and financial crises is still an open issue in economics and finance [1, 2]. There is serious debate on the nature of financial crises. The equilibrium school believes in a self-stabilizing market and attributes external shocks as the only source of cycles and crises [3-7], while the disequilibrium school mainly considers various mechanisms of market instability [8-10]. Current econometric analysis has difficulty in diagnosing crises since its analytical foundation is static probability distribution. The equilibrium school assumes the Gaussian distribution or i.i.d. with a finite mean and variance [4], while the disequilibrium school introduces non-Gaussian distributions, such as the Levy distribution, fat tails and power law [7, 11-13]. The disequilibrium approach is attractive in empirical analysis but impotent in policy studies, since it has little means to prevent external shocks. The third approach is a computational simulation of heterogeneous agents [14]. This approach can generate unstable patterns suggested by behavioral economics, but is hard to apply in empirical analysis of a financial crisis.

Our research strategy is to choose a proper degree of abstraction, so that it is simple enough to explain key empirical observations but general enough to integrate existing theories. We have solid evidence that financial movements are nonlinear and non-stationary in nature [2]. Therefore, we develop a general approach in analyzing a non-stationary time series, so that the equilibrium, stationary, and linear scenario are special cases of non-equilibrium, non-stationary, and nonlinear situations. Historical features can be used to test crisis theory [15]. Our breakthrough in diagnosing financial crisis is achieved by replacing the static model of the representative agent by the time-varying non-Gaussian probability distribution of population dynamics. The master equation approach is widely used in statistical mechanics in physics and

chemistry, which is called the social dynamics in studies of interacting agents in sociology and economics [16-19]. According to this approach, equilibrium analysis mainly considers the first (mean value) and second moment (variance), while non-equilibrium situations study social behavior with higher moments [20].

In a recent paper, we show that high moments representation can provide effective warning signals of a regime-switch or an upcoming financial crisis [21]. We will further demonstrate that the changing patterns in transition probability can diagnose the nature and cause of the 2008 financial crisis. Methodologically speaking, the time-varying probability distribution is richer than the static feature from the representative agents, but simpler than the computational model of heterogeneous agents in empirical and theoretical analysis. Time-varying probability distribution can be described by non-linear transition probability in a master equation for the birth-death process. The transition probability in different historical time windows provides valuable information on structural stability of dynamical markets, while the high moments representation may provide timely warning of coming crisis.

The technical issue is how to derive the transition probability from empirical observation and economic mechanisms since the Gaussian type distribution in equilibrium physics may not be valid for social systems [16, 22]. For example, the herd behavior in a social population may generate a bi-modular distribution [23, 24, 25]. Here, we do not make any ad hoc behavioral assumptions for market dynamics. Instead, we take a phenomenological approach to derive the transition probability from empirical data. We develop a numerical algorithm to establish a link between the master equation and the empirical estimation of transition probability. Herd behavior and bi-modular distribution can be explained by logistic interaction in transition probability [2, 24].

We estimate the transition probability in two separate periods: one period, 1950 to 1980, was dominated by Keynesian policy and New Deal regulation; and the period of 1981 to 2010 is the liberalization era started by the Reagan-administration and

includes the 2008 crisis. In each period, we assume that the transition probability is the function of market states (i.e. current prices), but independent of time. This procedure is similar to the two-stage econometric analysis. The difference is attributed to different mathematical representations. Econometric analysis is based on a matrix, while a probability distribution needs to solve partial differential equations. A more advanced mathematical representation may reveal more patterns in complex dynamics.

Through empirical analysis, we introduced a quantitative indicator of population behavior: the transition probability between neighboring states of price indexes. We discovered the nonlinear shape of transition probability for the period of liberalization and crisis, which is rooted in trading behavior. We found a visible link between the liberalization policy and the financial crisis. This result is very different from the exogenous school [7, 11, 13].

Through theoretical modeling, we demonstrate that the birth-death process is the proper model in population dynamics, which is capable of explaining all three observed features. The birth-death process originated in molecular dynamics in physics and has been introduced to describe an up-down process in the stock price movement [26-28]. We use the birth-death process as a unified model of linear (calm) and nonlinear (turbulent) markets. By means of moment expansion, we estimated the condition of the market breakdown, which is remarkably close to the real event. Unlike the model of heterogeneous agents, our population model of identical agents provides an alternate picture of animal spirits. Mass psychology is visualized by the rising and falling market tide that is measured by the net daily change rate.

Based on these findings, we get a new understanding of old conflicting thoughts. The so-called efficient market provides a simplifying linear picture of the calm market, whose higher moments are much smaller than the variance. The nonlinear turbulent market during the crisis resulted from the rise of high moments when the buy and sell pattern is remarkably nonlinear and asymmetric. There is strong evidence

of endogenous instability, since the financial market is resilient under repeated cycles and crisis, which is characterized by the stable regime of the relative deviations [2, 29, 30]. Our picture greatly extends the scope of equilibrium models in finance, which can be considered as a special case of a calm market in our nonlinear model.

2. Theoretical Models: The Master Equation and the Birth-Death Process

2.1. The Master Equation

The master equation is widely used in physics, chemistry, biology and finance [18, 31]. The change of the time-varying probability distribution is generated by the transitions from state x' to state x , minus the transitions from state x to state x' . The resulting master equation is given by the following partial differential equation:

$$\frac{\partial}{\partial t} P(x, t) = \int dx' [W(x | x', t) P(x', t) - W(x' | x, t) P(x, t)]. \quad (0)$$

$P(x, t)$ is the probability distribution function, $W(x | x', t)$ is its transition probability, represents the probability for the stochastic variable changing from state x' to x in time interval of t to $t + dt$. For the stationary case, $\frac{\partial}{\partial t} P(x, t) = 0$.

A stochastic process can be defined by equation (0) by introducing a specific transition probability.

2.2. The Birth and Death Process

The master equation of the birth-death process in the discrete price space is the following:

$$\begin{aligned} \frac{\partial P(x,t)}{\partial t} = & W_+(x-1)P(x-1,t) + W_-(x+1)P(x+1,t) \\ & - [W_+(x) + W_-(x)]P(x,t), \end{aligned} \quad (2)$$

Where $W_+(x) = W(x+1|x)$, $W_-(x) = W(x-1|x)$,

The birth-death process Eq. (2) is simpler than the master equation (0) because its state space is in discrete form. The unit of price is the minimal accounted change of the variable, which is 0.01 point as the unit for a stock index, and $\frac{0.01}{Y}$ for log-index $S = \ln Y$.

Transition probability can be described by a vector projected in a non-orthogonal set. Its basis function is $\{f_n : n \in \bullet\}$ with $f_0 = 1$, $f_1 = x$, and $f_n = \prod_{i=0}^{n-1} (x-i)$ for $n \geq 2$. Therefore, transition probability W can be expressed by a vector $(a_0, a_1, \dots, a_n)$ in space $(f_0, f_1, \dots, f_n)$. The transition probability can be derived from empirical data. We will discuss it in Section 3.

2.3. The Linear BD Process

The linear birth-death process, BD Process, is the simplest case of equation (2) when the coefficients of transition probability are: $a_1 \neq 0$ and $a_i|_{i \neq 1} = 0$. If the transition probability is linear with an identical birth rate and death rate, the birth-death process will converge to a Brownian motion with a detailed balance. Under the detailed balance, Cox and Ross [26] derived the Black-Scholes model from the birth-death process.

For the linear birth-death process:

$$W_+ = bx, \quad W_- = dx \quad (3)$$

Here, b is the birth (price-up) rate, and d is the death (price-down) rate for the linear birth-death process.

Price movements in a growing market can be visualized by the up and down dynamics driven by positive feedback ($W_+ = bx$) and negative feedback ($W_- = dx$) trading strategies. The trend emerges as an aggregate result of mass trading. The linear birth-death process produces a linear deterministic trend when $b - d > 0$.

$$\frac{dE(x(t))}{dt} = (b - d)E(x(t)) \quad (4)$$

We introduce a useful measurement of the relative deviation (RD), which is the ratio of standard deviation to its mean [2]:

$$RD = \frac{\text{standard deviation}}{\text{mean}} \quad \text{when mean} > 0 \quad (5)$$

We calculate the RD for two special cases: (a) when its time limit tends to zero. (b) when its time limit tends to infinity. Their solutions are:

$$\lim_{t \rightarrow 0} \Omega_{BD} = \sqrt{(b + d)t} \quad (6a)$$

$$\lim_{t \rightarrow \infty} \Omega_{BD} = \sqrt{\frac{b + d}{b - d}} \quad (6b)$$

From equation (6a) and (6b), we can see that the short-term perspective of the linear birth-death process is a diffusion process with an explosive relative deviation, while its long-term perspective is convergent to a steady state with a constant relative deviation. This result provides a good argument for using the birth-death process, which is capable of explaining the observed stable pattern of relative deviations in

stock and macro indexes [29, 30]. The proofs of (4) to (6) are given in the Appendix.

2.4. The Nonlinear Birth-Death Process with High Moment Expansion

In order to study market instability and financial crises, we study a nonlinear birth-death process with the 4th power of state x for mathematical simplicity.

We consider a nonlinear birth-death process; its transition probability has the following form:

$$\begin{aligned} W_+ &= b_0 + b_1 f_1 + b_2 f_2 + b_3 f_3 + b_4 f_4 \\ W_- &= d_0 + d_1 f_1 + d_2 f_2 + d_3 f_3 + d_4 f_4 \end{aligned} \tag{7}$$

Where $f_0 = 1$, $f_1 = x$, $f_2 = x(x-1)$, $f_3 = x(x-1)(x-2)$, and $f_4 = x(x-1)(x-2)(x-3)$.

Theoretically speaking, this formulation is a phenomenological description of nonlinearity up to the third power of x . Intuitively, the f function can be visualized as a consecutive trade up to four steps in a time interval by identical traders. We will see that the small nonlinearity of the 4th power to x is capable of understanding the mathematical condition of a market break-down or crisis.

It is hard to find an analytic solution with non-linear transition probability. But we can explore an approximation solution by means of high moment expansion and obtain the Fokker-Plank equation with Poisson Representation [32] for the high moments:

$$\begin{aligned}
\frac{\partial F}{\partial t} = & -\frac{\partial}{\partial \alpha}[(b_1 - d_1)\alpha + (b_2 - d_2)\alpha^2 + (b_3 - d_3)\alpha^3 + (b_4 - d_4)\alpha^4]F(\alpha, t) \\
& + \frac{\partial^2}{\partial \alpha^2}[b_1\alpha + (2b_2 - d_2)\alpha^2 + (3b_3 - 2d_3)\alpha^3 + (4b_4 - 3d_4)\alpha^4]F(\alpha, t) \\
& - \frac{\partial^3}{\partial \alpha^3}[b_2\alpha^2 + (3b_3 - d_3)\alpha^3 + (6b_4 - 3d_4)\alpha^4]F(\alpha, t) \\
& + \frac{\partial^4}{\partial \alpha^4}[b_3\alpha^3 + (4b_4 - d_4)\alpha^4]F(\alpha, t) \\
& - \frac{\partial^5}{\partial \alpha^5}b_4\alpha^4F(\alpha, t)
\end{aligned} \tag{8}$$

where F is no longer a real measure for high order moments. It implies that the stationary distribution may not exist. We have showed the dynamics of a nonlinear turbulent market when high moments diverge [21]. The theoretical moments would diverge at the point of x when:

$$\frac{\partial}{\partial x}[(b_1 - d_1)x + (b_2 - d_2)x^2 + (b_3 - d_3)x^3 + (b_4 - d_4)x^4] = 0, \tag{9}$$

since the left side of Equation (9) would appear in the denominator when solving Equation (8) using the perturbation method [32].

3. Transition Probability Estimated from Empirical Data

3.1. Estimating Transition Probability in Two Periods

In theory, transition probability is always changing over time. This imposes a tremendous difficulty in the empirical analysis of transition probability. In econometric analysis, we can divide the available time series into several periods, and assume that the structural form is not changing over time within each period. Similarly, we may assume that the transition probability in each period is only a

function of a price state, not the function of time. We chose 1980 as the dividing line since President Ronald Reagan initiated the liberalization era in market deregulation at that time. We wish to observe the structural change between the periods of 1950-1980 and 1981-2010.

In empirical analysis, the data frequency restricts the resolution of trading behavior. Given the frequency of data set $\{x_t | t \in [0, +\infty)\}$ (length of Δt between two successive point x_{t+1} and x_t), only the net aggregate result of all trades during Δt accounts for the transition probability. For example, if we use the daily data, the balanced buyer-initiated and seller-initiated trades within any interval during the day have no influence on the moments calculated on a daily basis.

Assume that the transition probability doesn't vary within the specific period. If there are N samples at a given value x^0 , among which n_+ samples move up by the average jump magnitude Δx_+ and n_- samples move down by the average jump magnitude Δx_- in the next day, the transition probabilities at x^0 are

$$W(x^0 + 1 | x^0) = \frac{n_+}{N} \Delta x_+, \text{ and } W(x^0 - 1 | x^0) = \frac{n_-}{N} \Delta x_-$$

Therefore, $\frac{n_+}{N}$ and $\frac{n_-}{N}$ is the probability of moving up and down, Δx_+ and Δx_- are the respective numbers of “standard” trades. We calculated the transition probability from the S&P 500 daily index. Their pattern for the period of 1981-1996 and for the period of 1997-2010 is shown in Fig. 1 and Fig. 2 respectively.

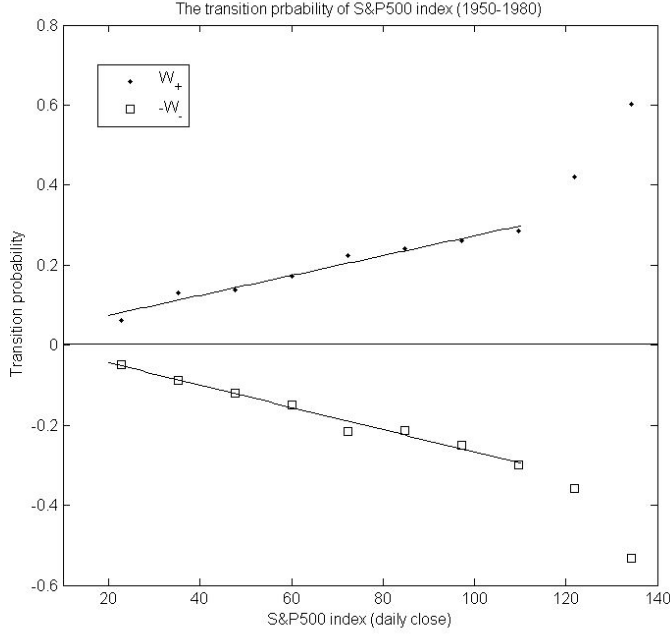


FIG.1. The transition probabilities (W_+ and $-W_-$) of the S&P 500 daily close in 1950-1980. The horizontal axis is the price level of the S&P 500 daily index. The main curves of W_+ and $-W_-$ (except the segments between 110 to 140 points) are not far from linear (the straight linear fitting lines). In fitting the transition probability, the tops (higher than 110) of several waves were taken out because of too few data points available at that range.

Here, the diverging curves are caused by growing trend of stock index. The upper curve can be explained by the “strength” with positive trading strategy, and the lower curve the strength with negative trading strategy. Intuitively, net price movements are resulted from the constant battles between the “Bull camp” and the “Bear camp”.

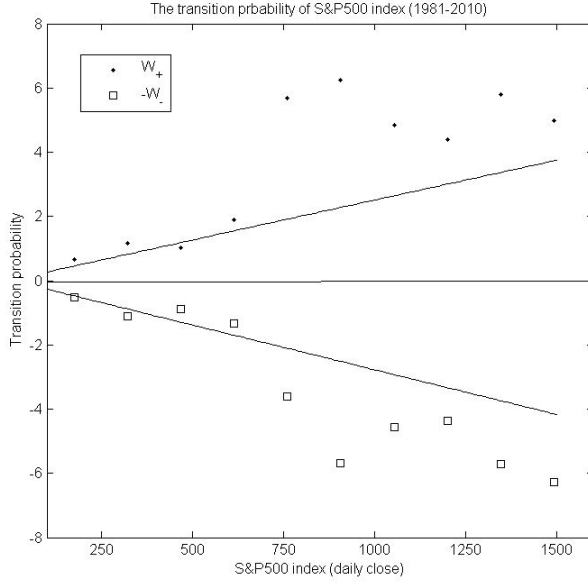


FIG.2. In 1981-2010, the transition probabilities (W_+ and $-W_-$) of the S&P 500 index are marked by two curves, which are highly non-linear with two visible humps or dips. The two straight lines here are the extensions of the straight fitting lines in FIG 1.

Comparing FIG.1 with FIG.2 we have three observations:

First, the transition probability has two curves: both curves are not straight lines. It is a clear feature of nonlinear dynamics in the birth-death process. The upper curve indicates a price-up magnitude driven by positive feedback and the lower curve indicates a price-down magnitude driven by negative feedback in market trading. The observed behavior is more complex than the noise trader model [10].

Second, the upper and lower curves are not symmetric, since there is a growth trend in the market index time series.

Third, there is remarkable difference between the two periods. For period I, 1980-1996 without severe crisis, the two transition probability curves are more or less balanced with only gradual changes. For period II, 1997-2010 with the 2008 crisis, the transition probability curves have visible humps in the upper curve and dips in the lower curve.

Fourth, we may define three dynamic regimes based on the different patterns from transition probability. The artificial linear lines represent the ideal image of the so-called efficient market. The quasi-linear pattern of transition probability in FIG. 1 describes a (stable)calm market without a crash or crisis. The nonlinear pattern in FIG. 2 implies an (unstable)turbulent market with a crash and crisis. Both the terms “calm” and “turbulent” markets are used to characterize social psychology in behavioral finance.

The observable patterns in FIG. 1 and FIG 2 can be shown by standard regression analysis. In FIG.1, linear model can explain over 96% of the curves, which are 96.5% ($R^2 = 0.9649$) for the positive transition probability (the upper curve, $W_+ = 0.002482x + 0.02513$) and 98% ($R^2 = 0.9828$) for the negative transition probability (the lower curve, $-W_- = -0.002788x + 0.01151$). In FIG. 2, linear regression can only explain 65% of the positive transition probability ($R^2 = 0.6511$) for the positive upper curve ($W_+ = 0.004038x + 0.3019$), and 86% ($R^2 = 0.8573$) for the negative transition probability ($-W_- = -0.004694x + 0.5010$). In FIG. 1, the linear regression coefficients are much smaller than which in FIG. 2. This implies that the variance of daily percentage index changes has grown larger since 1981. During 1997-2010, linear regression can only explain 20% ($R^2 = 0.1953$) of the positive transition probability, and 7% ($R^2 = 0.073$) of the negative transition probability. Nonlinearity represents main segments of the transition probability curves.

Similarly, the transition probability of Dow-Jones Industrial index (DJI) shows stronger non-linearity since 1981, which is shown in FIG. 3.

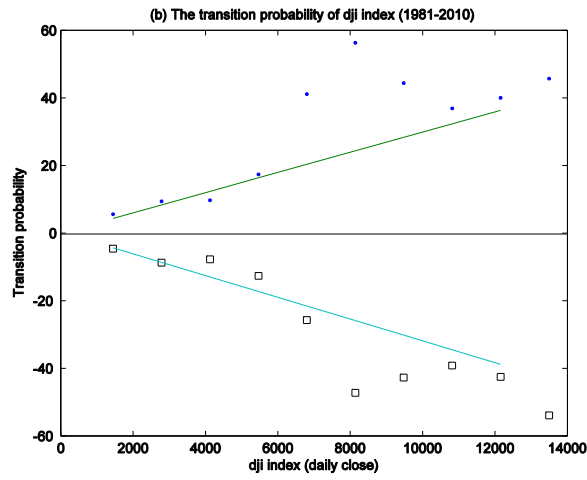
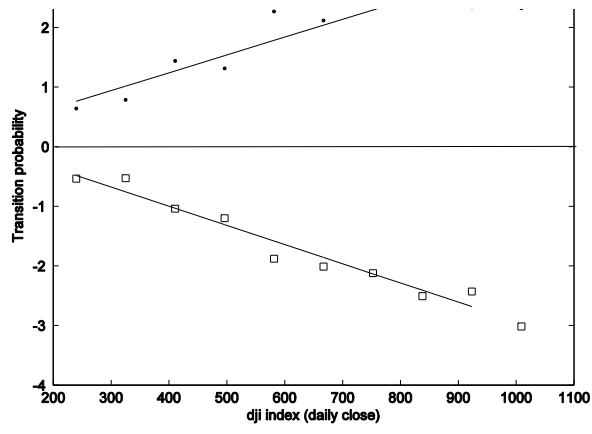


FIG.3. The transition probabilities (W_+ and $-W_-$) of the DJI(Dow Jones Industrial) index daily close in 1950-1980 [figure (a)] and 1981-2010 [figure (b)]. The horizontal axis is the price level of the DJI daily index. In figure (a) the main curves of W_+ and $-W_-$ are not far from linear (the straight linear fitting lines). In fitting the transition probability in (a), the tops (higher than 950) of several waves were taken out because of too few data points available at that range. In 1981-2010 [figure (b)], the transition probabilities (W_+ and $-W_-$) of the DJI index are marked by two curves, which are highly non-linear. The two straight lines in figure (b) are the extensions of the straight fitting lines in figure (a).

In FIG. 3(a), linear fitting can explain over 85% of the curves, which are 86% ($R^2 = 0.8578$) for the positive transition probability (the upper curve,

$W_+ = 0.002986x + 0.04683$) and 95% ($R^2 = 0.9471$) for the negative transition probability (the lower curve, $-W_- = -0.003214x + 0.2851$). In FIG. 3(b), linear regression can only explain 65% of the positive transition probability ($R^2 = 0.6534$) for the positive upper curve ($W_+ = 0.004088x + 1.176$), and 82% ($R^2 = 0.8241$) for the negative transition probability ($-W_- = -0.004326x + 3.739$). During 1997-2010, linear regression can only explain 34% ($R^2 = 0.3429$) of the DJI positive transition probability, and 0.5% ($R^2 = 0.0045$) of the negative transition probability.

We can see the behavior changes that occurred within some price ranges. The polarized pattern appears in public opinion on future market trends [24]. The psychological source of the market instability was characterized as “animal spirits” [33, 34].

3.2. Estimating Transition Probability Coefficients

The following simplified equations are used to calculate the transition probability from $Y(t)$ to estimate the coefficients of W_+ and W_-

$$\begin{aligned} W_+ &= b_0 + b_1Y + b_2Y^2 + b_3Y^3 + b_4Y^4 \\ W_- &= d_0 + d_1Y + d_2Y^2 + d_3Y^3 + d_4Y^4 \end{aligned} \tag{10}$$

The coefficients of W_+ and W_- from the S&P 500 index during 1997-2010 is shown in Table 1.

Table 1. The coefficient of transition probability of the S&P 500 Index (original daily close) for the period of 1997-2010

The polynomial fit coefficient					R^2
$b_0 = -101.4$	$b_1 = 0.5011$	$b_2 = -8.032 \times 10^{-4}$	$b_3 = 5.368 \times 10^{-7}$	$b_4 = -1.284 \times 10^{-10}$	0.9517
$d_0 = -625.9$	$d_1 = 2.277$	$d_2 = -3.023 \times 10^{-3}$	$d_3 = 1.749 \times 10^{-6}$	$d_4 = -3.725 \times 10^{-10}$	0.9432

In Table 1, the period of 1997-2010 was chosen for mathematical simplicity of curve fitting. We will show the transition probability faithfully recorded in the recent subprime crisis in the next section.

3.3. Estimating the Condition of Market Break-Down

We refer to the market as turbulent when equation (9) is valid and the probability distribution no longer exists. This situation is similar to critical fluctuation in statistical physics [32].

Under this situation, we have four implications: First, all statistic variables become meaningless. Second, the market trend collapses. In other words, market expectations have no consensus, only panic rules the market. Third, the condition (8) reveals the possibility of market breakdowns. Fourth, the crisis region is beyond the scope of the nonlinear birth-death process. We need a more advanced tool to describe the dynamic process in an unstable turbulent market. This knowledge is absent in linear models of finance theory.

The nonlinear pattern of transition probability reveals the dynamic trend of population dynamics. Since we know the coefficients of transition probability, we could estimate the location of the market crisis. If the positive coefficients represent the degree of positive feedback and the negative coefficients the degree of negative

feedback, the difference between positive and negative feedback measures the net movement within a day, we refer to it as the net daily change rate.

Keynes and behavioral economists pointed out the role of mass psychology [33, 35]. We can visualize the market tide driven by collective psychology as a curve. Its falling segment represents a market tide towards equilibrium while a rising segment signals a market tide towards disequilibrium. FIG. 4 shows the numerical results of the net daily change rate, which indicates down – up – down market tides. The up phase describes a collective fad for a market bubble. We assume that the turning point from the up to the down phase may generate a market breakdown or crisis, and therefore a critical point of financial crises. The critical behaviors of crises have also been characterized by high moments diverges and extraordinarily large trading volumes [21, 36, 37].

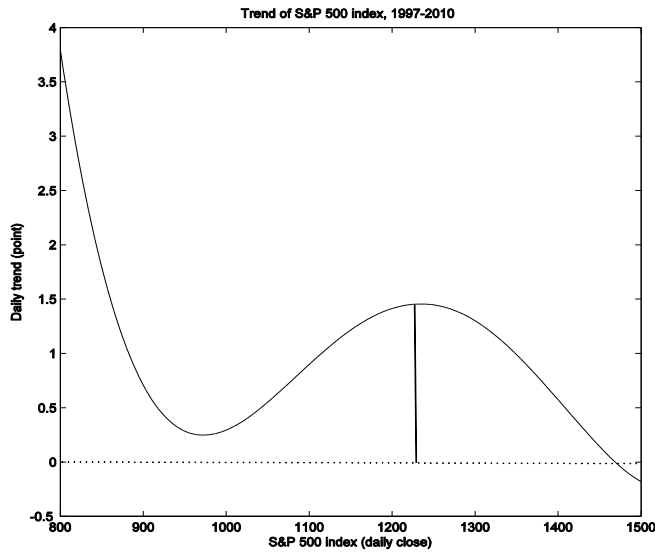


FIG 4. The curve of a changing market tide in terms of the net daily change rate (1997-2010). The curve is calculated from the 4th degree polynomial fitted transition probability from 1997-2010. The up segment indicates two hot-speculation periods from 950 to 1229 in 1997-2000 and 2003-2008, and panic in 2008. The vertical line marks the turning (critical) point in market psychology, which is 1229 in the price level.

From FIG 4, the turning point can be estimated from the numerical solution of the nonlinear birth-death process. We have a quantitative evidence to infer that the stock market break-down may happen near the 1229-th point. According to historical data, the S&P 500 index closed at 1209.13 on 25-Sep-2008, when the Office of Thrift Supervision (OTS) seized Washington Mutual, and sold its banking assets to JP Morgan Chase for \$1.9 billion. This event was the peak from a chain events preceding the 2008 financial crisis. Before this event, Fannie Mae and Freddie Mac were nationalized by the U.S. government, Lehman Brothers bankrupted, AIG encountered a liquidity crisis. Then, the stock market went to panic since 26-Sep-2008. Effectively, our estimation of historical turning point provides an accurate indicator for coming crisis in addition to high moments approach in crisis warning [21].

Certainly, transition probability can only be estimated ex post. In practice, the rising tide signals an upcoming bubble in the financial market. This is a valuable indicator for an early warning of a crisis.

For empirical observation, an unstable turbulent market can be observed from high frequency data with a sudden rise of trading volume or even a market frozen in a period of hours or days. But a turbulent market may not exist long when government intervention calms the market in a modern economy. Our approach is capable of identifying the period of market turbulence, which creates the space for government interference.

4. Conclusion

There is a widely perceived image that neoclassical economics imitates equilibrium physics [38]. This is partially true in a philosophical paradigm because both of them belong to the equilibrium school of theoretical thinking, but not true in the mathematical formulation for empirical analysis. For example, the Brownian

motion model in physics is molecule dynamics with many particles, but option pricing based on geometric Brownian motion is a representative agent model with only one particle [39, 40]. Statistical mechanics starts with infinite moments with time-varying distribution probability, but econometric analysis is based on the i.i.d. model with a finite mean and variance. Most scientists agree that economic phenomena are more complex than physics and chemistry. Strangely, economic models are much simpler than models in physics and chemistry. Paul Krugman, the 2008 Nobel Laureate in economics, recently wrote a provocative article in the New York Times, titled “How Economists Got It So Wrong?” His answer was “mistaken beauty for truth” [41]. However, there are no scientific criteria to assert that linear models are prettier than nonlinear ones [42]. The fundamental issue here is not in the mathematical beauty but the empirical relevance of economic theory. We know that existing linear models are not capable of explaining market turbulence and financial crisis. To paraphrase Krugman, we demonstrate that equilibrium models in finance are “mistaken simplicity for complexity” by ignoring nonlinear dynamics, high moment deviations, time-varying distribution, and social interactions. Our progress was made by integrating these complex factors into properly formulated economic dynamics. Our inspiration came from Einstein, when he discovered that non-Euclidean geometry was more relevant for general relativity than Euclidean geometry. A general framework with more advanced mathematical representations is needed to solve old controversies in economics.

In this article, we show that the master equation approach developed in statistical mechanics can be applied to study macro and financial dynamics [16], if transition probability can be extended from a Gaussian distribution to a logistic-type nonlinear function [24, 29]. We found that the time-varying probability distribution and the birth-death process are useful in diagnosing the nature of business cycles and financial crisis.

From empirical transition probability, we found that both negative-feedback and

positive-feedback trading behavior coexists in transition probability. This picture is different from equilibrium economics and behavioral finance, since the former only considers the stabilizing role of negative feedback and the latter emphasizes the destabilizing role of positive feedback. The empirical patterns of transition probability reveal two additional dynamic regimes in addition to the linear regime in price dynamics: the quasi equilibrium process (calm market) when positive and negative transition curves are smooth and near symmetric; the disequilibrium process (turbulent market) when its positive and negative transition curves are S-shaped and significantly asymmetric. Using two-period analysis of empirical data, we found a clear link between liberalization policy and financial crisis. Its policy implication is very different from the exogenous models, such as power law, fat tails, and the Black Swan theory.

The birth-death process is simple enough to explain the origin of a viable market with persistent fluctuations even during a crisis period: the stable pattern of the long-term relative deviation is an inherent nature of the population model. The nonlinear birth-death process is a general framework that integrates the linear calm market regime in the equilibrium school and the nonlinear turbulent market regime in the disequilibrium school. Its nonlinear pattern of transition probability demonstrates the nonlinear nature of endogenous instability in business cycles. Its theoretical power is demonstrated by the fact that our numerical estimation of the critical point is close to the historical event during the U.S. 2008 financial crisis. [The spontaneous regime switch and critical point](#) was also observed from the [real time simulations of the endogenous noise model of population dynamics](#) [43].

Philosophically speaking, the so-called efficient market in finance literature is essentially a simplified linear model of the calm market, which has three observable features: First, the growth trend from stock market indexes can be ignored in the short-term perspective in financial econometrics. Second, the higher moments are quite small in comparison with the variance. Therefore, price movements can be

considered as a diffusion process with a Gaussian distribution or i.i.d. Third, the diffusion process is a special case of the linear birth-death process with a short-term perspective, whose relative deviation is explosive. It implies that the diffusion process, i.e. in the form of the pattern of the stable relative deviation [2, 29, 30, 44]. However, the noise trader, the random walk model or the geometric Brownian motion is not capable of describing a “viable market” with an observed model in behavioral finance. The model with two discrete periods when a positive feedback and negative feedback trading strategy is essentially a stepping-stone for our nonlinear model [10]. These two trading strategies can be considered as special cases in our nonlinear birth-death process.

The policy implications from our analysis are self-evident. Market bubbles and financial crisis occurred in the period of 1981-2010, which reveals the possible link between the liberalization policy started by President Reagan and the 2008 financial crisis. Clearly, the price level alone is not enough to gauge market sentiment. Regulating market leverage in trading would be more effective in preventing a possible crisis.

Methodologically speaking, the population dynamics of the birth-death process provides a useful framework for studying complex financial dynamics including calm and turbulent markets as well as market resilience after a crisis. Compared to existing approaches in parametric econometrics and computational simulations based on heterogeneous agents, transition probability provides an effective approach both in the empirical analysis and theoretical understanding of market instability and financial crisis. Our population model of identical agents provides a simpler explanation of market bubbles than the heterogeneous agent model. Nonlinearity and population behavior play key roles in the genesis of a financial crisis, which is beyond the scope of linear dynamics and representative agent models. We are developing a general model of option pricing based on the master equation and birth-death process. We will address this issue elsewhere.

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APPENDIX: THE RELATIVE DEVIATION OF THE LINEAR BIRTH-DEATH PROCESS

A linear birth-death process can be simply approximated by a deterministic process $E(x(t))$ plus a stochastic process z [45]. The deterministic trend is

$$\frac{dE(x)}{dt} = W_+[E(x)] - W_-[E(x)] \quad (\text{A1})$$

and the Fokker-Plank equation for variance is

$$\begin{aligned} \frac{\partial P(z, t)}{\partial t} = & -(W'_+[E(x)] - W'_-[E(x)]) \frac{\partial}{\partial z} zP(z, t) \\ & + \frac{1}{2} \frac{\partial^2}{\partial z^2} (W_+[E(x)] + W_-[E(x)]) P(z, t). \end{aligned} \quad (\text{A2})$$

Where

$$\langle z(t), z(t) \rangle = \int_0^t dt' (W_+[E(x(t'))] + W_-[E(x(t'))]) \exp \left\{ 2 \int_{t'}^t (W'_+[E(x(s))] - W'_-[E(x(s))]) ds \right\}$$

is the variance of x_t . Hence the volatility σ_t^2 has a value evolves with the expectation $E(x(t))$

$$\sigma_t^2 = \frac{b+d}{b-d} E(x(t)) [e^{(b-d)t} - 1] \quad (\text{A3})$$

Where we set $E(x(0)) = 1$, b is a linear $W_+[E(x)]$, d is a linear $W_-[E(x)]$, $b > d$.

Then the RD of linear BD process is $\Omega_{BD} = \sqrt{\frac{b+d}{b-d}} [1 - e^{-(b-d)t}]$, which has three special cases:

$$\begin{aligned}\lim_{t \rightarrow \infty} \Omega_{BD} &= \sqrt{\frac{b+d}{b-d}}, \\ \lim_{t \rightarrow 0} \Omega_{BD} &= \sqrt{(b+d)t}, \\ \lim_{b \rightarrow d} \Omega_{BD} &= \sqrt{(b+d)t}.\end{aligned}\tag{A4}$$

Note that here Ω_{BD} is the RD for $Y(t)$. For logarithmic series S , $\lim_{t \rightarrow 0} \Omega_{BD}(S)$ and $\lim_{b \rightarrow d} \Omega_{BD}(S)$ are also growing with $\sqrt{t}$. And if $\Omega_{BD}(Y)|_{t \rightarrow 0}$ is stable, empirical $\Omega_{BD}(S)|_{t \rightarrow 0}$ is stable with only a slight decrease (the proof of the similarity of $\Omega_{BD}(Y)$ and $\Omega_{BD}(S)$ will be published elsewhere). Therefore, the linear Birth-death process can describe the stable RD in both original and logarithmic economic indexes.

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