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Searching for Economic Chaos: A Challenge to Econometric Practice and Nonlinear Tests

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Chaos research has attracted wide interest in the scientific community. Convincing empirical evidence for it has been found in fluid dynamics (Brandstater and Swinney, 1987), chemistry (Argoul et al., 1987), and biology (Guevara, Glass, and Shrier, 1981). Relatively less convincing reports come from epidemiology, population dynamics, meteorology, and astronomy (Pool, 1989). Evidence for it in economic data has been published in my own work with Barnett (Chen, 1987b, 1988a,b; Barnett and Chen, 1987, 1988) and in others including Brock and Sayers (1988) and Scheinkman and LeBaron (1989). This work is still controversial.

Empirical studies of economic chaos began in mid–1980 (Chen, 1984, 1987b; Sayers, 1985; Brock, 1986; Barnett and Chen, 1987, 1988). Nonlinearity (Ashly, Patterson, and Hinich, 1986; Brock and Sayers, 1988; Scheinkman and LeBaron, 1989), nonnormality (Ashly, Pattern, and Hinich, 1986), and nonindependence (Brock, Dechart, and Scheinkman, 1987; Hsieh 1989) in economic time series is widely discovered. Negative or "mixed" findings are also reported (Sayers, 1985,1989; Brock and Sayers, 1988; Frank and Stengos, 1988; Frank, Gencay, and Stengos, 1988). Little evidence of chaos is found in monetary indexes (Chen, 1987b, 1988a,b), daily stock returns (Chen, 1984; Scheinkman and LeBaron, 1989), and laboratory simulations (Sterman, Mosekilde, and Larsen, 1989). There is a fierce debate about the empirical findings of economic chaos (Chen, 1988b; Brock and Sayers, 1988; Ramsey, Sayers, and Rothman, 1990).

In this chapter I will first introduce some of the techniques for distinguishing between randomly generated data and data generated by deterministic processes. I then analyze pitfalls in statistical tests designed to detect chaos. My work on monetary aggregates serves as an example to discuss the problem of inference with economic time series and to illustrate the usefulness of the continuous time model. This model is just sufficient to generate behavior that closely resembles the data.

Distinguishing between Deterministic and Stochastic Processes

There are at least four possible candidates for describing fluctuating time series: linear stochastic processes, discrete deterministic chaos, continuous deterministic chaos, and nonlinear deterministic chaos plus noise. Testing and modeling the last one are only in its infancy, because a high level of noise will easily destroy the subtle signal of deterministic chaos. I discuss the first three candidates here and give numerical examples of white noise and deterministic chaos as the background for further discussions. They include the linear autoregressive AR(2) model, the discrete time logistic model, which is widely used in population studies and economics, and the continuous-time spiral chaos or Rossler model (1976).

Sample time sequences of these models are shown in Figure 15.1. They all exhibit irregular economic fluctuations very much like economic data when appropriate scales are used. However, a closer examination reveals differences among them.

How can we distinguish between such different theoretical specifications? Can we tell if a given economic time series is generated by one of them? These are the basic questions we shall discuss. Four main tools are available, the "phase portrait," the autocorrelation function, the Lyapunov exponent, and the fractal dimension. I shall describe them briefly in turn.

Phase Space and Phase Portrait

From a given time series $X(t)$, an m-dimensional vector $V(m,T)$ in phase space can be constructed by the m-history with time delay $T : V(m,T) = \{X(t), X(t+T), \ldots, X[t+(m-1)T]\}$, where m is the embedding dimension of phase space (Takens, 1981). This is a powerful tool in developing numerical algorithms of nonlinear dynamics, since it is much easier to observe only one variable than to analyze a complex multidimensional system.

The phase portrait in two-dimensional phase space $X(t+T)$ versus $X(t)$ gives a clear picture of the underlying dynamics of a time series. Figure 15.2 displays the phase portrait of the three models using the sample data of Figure 15.1. The nearly uniform cloud of points in Figure 15.2a closely resembles the phase portrait of random noise (with infinite degree of freedom). The curved image in Figure 15.2b is characteristic of the one-dimensional unimodal discrete chaos. The spiral pattern in Figure 15.2c is typical of a strange attractor whose dimensionality is not an integer. Its wandering orbit differs from periodic cycles.

Long-Term Autocorrelations

The autocorrelation function is another useful concept in analyzing time series. The autocorrelation function $AC(J)$ is defined by

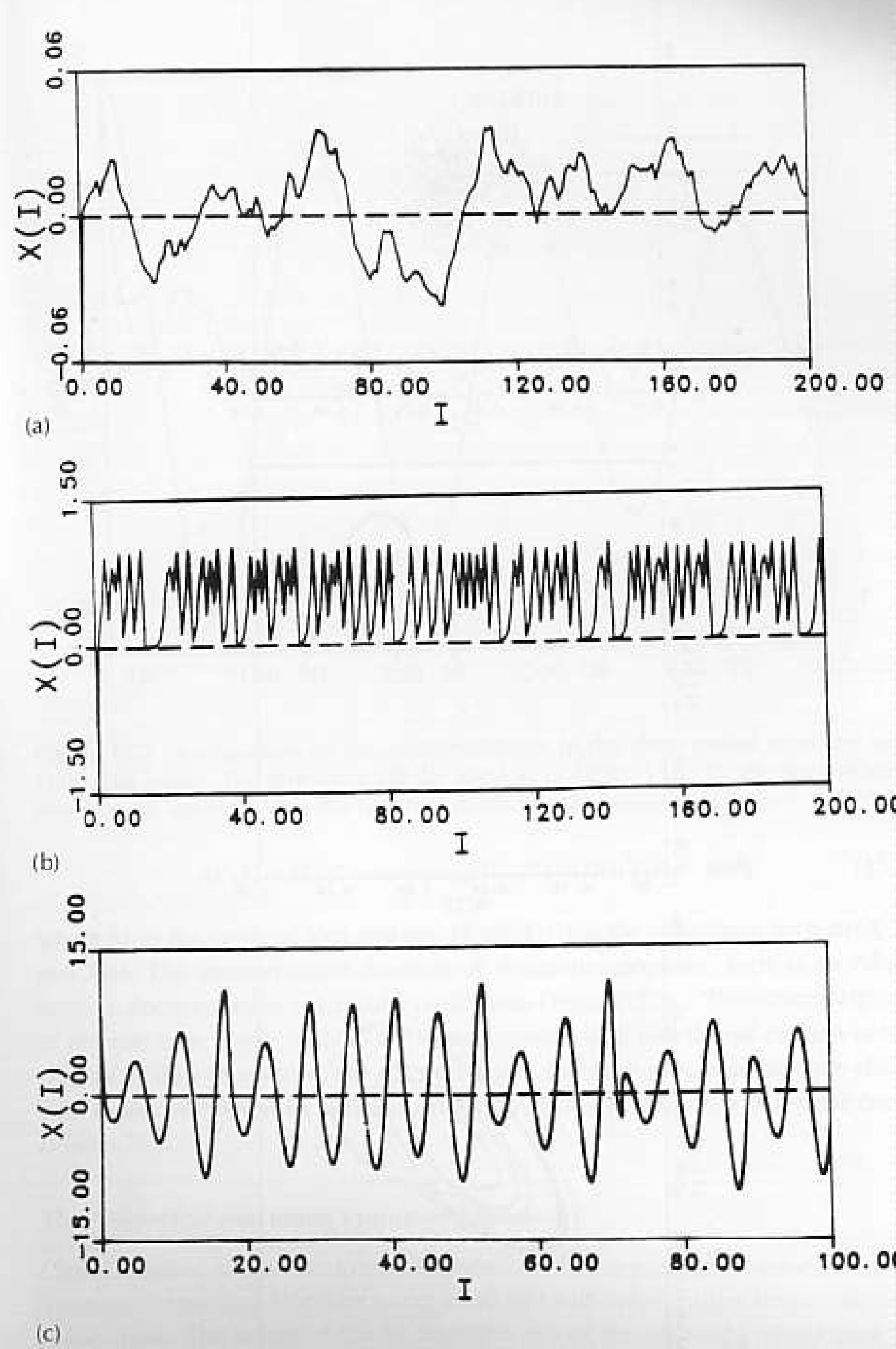


Figure 15.1 Comparison of the time series of model solutions. Their time units a arbitrary. (a) AR(2) linear stochastic model. (b) Discrete logistic chaos generated mapping $X(t+1) = 4X(t)[1 - X(t)]$. (c) Rossler model of spiral chaos with time interv $dt = 0.05$.

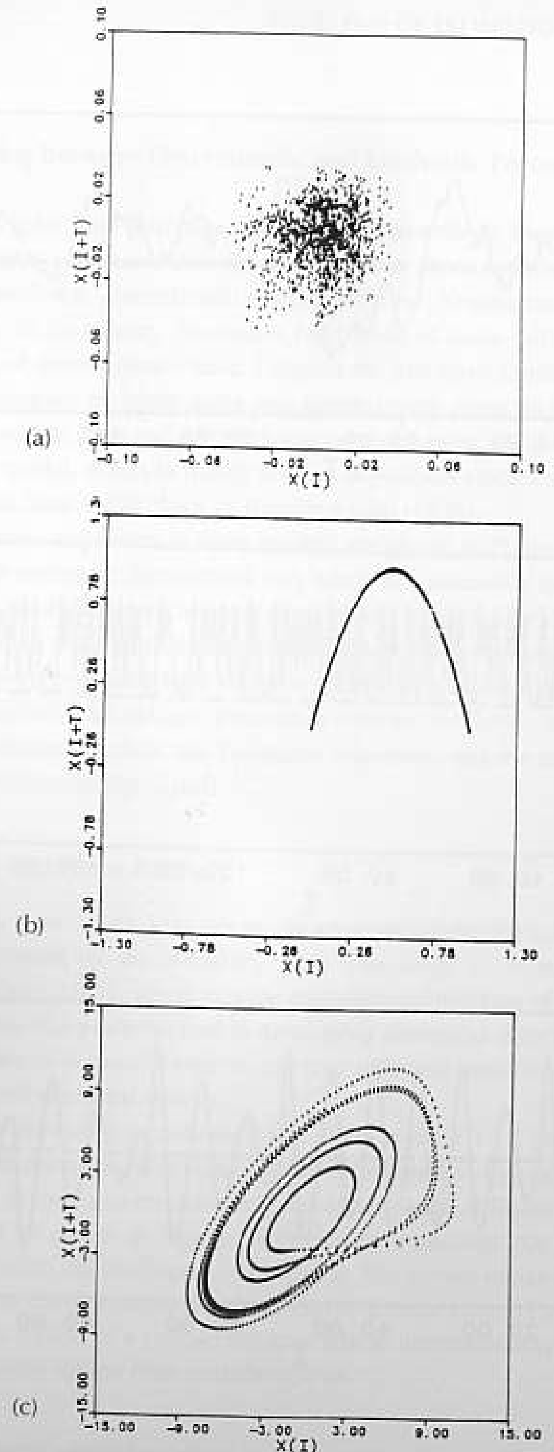


Figure 15.2 Comparison of the phase portraits of model solutions. $N = 1000$. (a) AR(2) model with $T = 20$. (b) Logistic chaos with $T = 1$. (c) Rossler model with $T = 1$ and $dt = 0.05$.

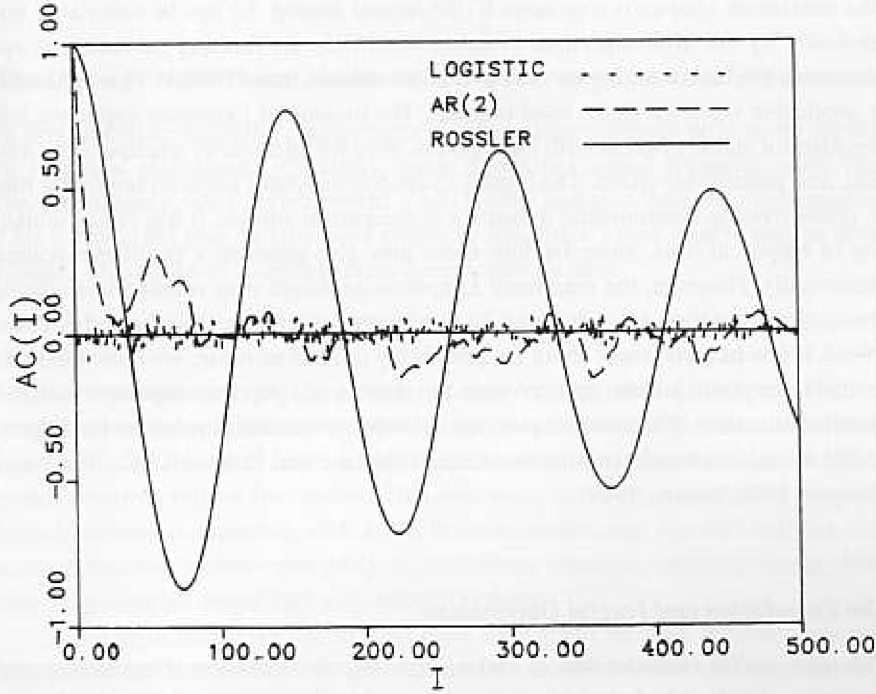


Figure 15.3 Comparison of the autocorrelations of the three model solutions wi 1000 data points. The time units are the same as in Figure 15.1. T_d, the decorrelatic time, can be determined by the first zero point of autocorrelation function.

$$AC(I) = AC(t' - t) = \mathrm{cov}[X(t'), X(t)]/E[(X(t) - M)^2] \qquad (15.$$

where M is the mean of $X(t)$ and cov $[X(t'), X(t)]$ is the covariance between $X($ and $X(t)$. The autocorrelation function of stochastic processes, such as an AR(process, decays quickly to irregular oscillations (Figure 15.3a). The autocorrelatio of discrete time chaos, such as the logistic model, look like that of random nois (Figure 15.3b). In contrast, the autocorrelation function of continuous-time chac such as that of the Rossler attractor, shows wave-like oscillations with smooth deca (Figure 15.3c).

The Numerical Maximum Lyapunov Exponent

Chaotic motion is sensitive to initial conditions. This sensitivity is measured by t Lyapunov exponents. Consider a very small ball with radius $\varepsilon_i(0)$ at time $t = 0$ in t phase space. The length of the ith principal axis of the ellipsoid evolved from t ball at time t is $\varepsilon_i(t)$. The spectrum of Lyapunov exponents λ_i from an initial poi can be obtained theoretically by (Farmer, 1982).

$$\lambda_i = \lim_{t\to\infty} \lim_{\varepsilon(0)\to 0} \{\ln[\varepsilon_i(t)/\varepsilon_i(0)]/t\} \qquad (15.$$

The maximum Lyapunov exponent λ (the largest among λ_i) can be calculated numerically by the Wolf algorithm (Wolf et al., 1985). Its limiting procedure is approximated by an averaging process over the evolution time EVOLV. This algorithm is applicable when the noise level is small. The maximum Lyapunov exponent λ is negative for stable systems with fixed points, zero for periodic or quasiperiodic motion, and positive for chaos. The largest Lyapunov exponent plays an important role in characterizing deterministic dynamics in theoretical studies. It has rather limited use in empirical tests, since random noise may also generate a positive exponent numerically. However, the maximum Lyapunov exponent may reveal some clue of chaos, if the order of $\lambda - 1$ is about T_d the decorrelation time (Nicolis and Nicolis, 1986). If one has any doubt about the possibility of random noise, we recommended to check the phase portrait and compare the reverse of Lyapunov exponent with the decorrelation time. These techniques can tell whether business cycles are likely generated by unit-root random process or chaos (Nelson and Plosser, 1982; Frank and Stengos, 1988; Sayers, 1989).

The Correlation and Fractal Dimensions

The most useful characteristic of chaos is its fractal dimension (Grassberger and Procaccia, 1984), which provides a lower bound to the degrees of freedom for the dynamic system. The popular Grassberger–Procaccia (GP hereafter) algorithm estimates the fractal dimension by means of the correlation dimension D (Grassberger and Procaccia, 1983). The correlation integral $C_m(R)$ is the number of pairs of points in m-dimensional phase space, whose distance between each other is less than R. For random or chaotic motion, the correlation integral $C_m(R)$ may distribute uniformly in some region of the phase space, and it has a scaling relation of R^D. Therefore, we have

$$\ln_2 C_m(R) = D \ln_2 R + \text{constant} \tag{15.3}$$

For white noise, D is an integer equal to the embedding dimension m. For deterministic chaos, D is less than or equal to the fractal dimension.

Pitfalls in Statistical Testing for Chaos

There are two major pitfalls in testing empirical data for chaos that need to be recognized. These involve (1) the common problems caused by insufficient information in empirical tests; and (2) the specific limitations of statistical inference for distinguishing chaotic from stochastic processes.

The Discrepancy between Mathematical Theory and Numerical Experime

In any scientific discipline, mathematical theory approximates only some asp of empirical phenomena. Certainly, more difficulties attend empirical work theoretical study, since the real world is much more complex than simplified mod This is especially true for studies of nonlinear dynamics. I now want to outline s of these difficulties in detecting chaos from empirical data.

Sparse Data

Typical experiments in physics, chemistry, and biology often collect tens of th sands to almost a million data points and sampling time usually cover more th hundred cycles. However, most economic indicators have only several hundred points covering only a few cycles. This deficiency prevents many of the powe tools in nonlinear dynamics, such as the Poincare return map, spectral analysis, tual information, saddle-orbit analysis, and others based on statistical theory, f detecting intrinsic irregularity even when it is there.

Some algorithms give useful hints even for a small data set, but their pow much reduced. Worse, they may generate numerical artifacts. For example, the a correlations of continuous-time chaos models in numerical models show expone decay when the time span is very large (Grossmann and Sonneborn-Schmick, 19 However, the autocorrelations of continuous-time chaos look like those of peri movements when only a few cycles of data are available (Figure 15.3c). Small sets will introduce spurious low frequency in the power spectrum when there only about hundreds of data points available (Nelson and Kang, 1981).

The problem of sparse data is especially acute in dimension calculations cause their data requirements are severe. The minimum number of data points required in dimension estimation has an exponential relation with the underl dimension D, i.e., $N_D = h^D$, where h varies with different attractors (Mayer-K 1986). For example, in the case of the Mackey–Glass model (Mackey and G 1977), the required data for $D = 2$ is $N_D > 500$ points; and that for $D = 3$ is N 10,000 points (Kostelich and Swinney, 1989). We also investigate the effect o sample rate in dimension calculation. Generally speaking, 10–100 points per c are needed for the Mackey–Glass model. The relative error of the numerical cor tion dimension is about 1% for 100 cycles, 3% for 30 cycles, 8% for 10 cycles 18% for only 5 cycles.

It is also found that a discrete map needs even more data points than a tinuous flow. For instance, the reasonable number of data points is 5000 for H attractor (Henon, 1976) with $D = 1.26$ (Ramsey and Yuan, 1989). As a rule of th the observed dimensionality in empirical data cannot be higher than 5 and em ding dimension in calculation should not be larger than 10, when data size is than 10,000.

Noise

The second major problem is that the subtle information of deterministic chaos can be contaminated by numerical or measurement noise. The question is: To what degree can noise be tolerated in empirical tests? There are several numerical tests in terms of uniform noise. For example, it is found that the phase portrait of the noisy Henon map can be recognized and the correlation dimension can be estimated when random jitter is chosen from [−0.05, 0.05], the up-bond of noise/signal ratio is 0.1% for the correlation dimension of Mackey–Glass model (Ben-Mizrachi, Procaccia, and Grassberger, 1983). In estimating the largest Lyapunov exponent, the allowance is 5% (Wolf et al., 1985).

Continuous and Discrete Time and the Time Unit

For qualitative models in economic theory, the choice between difference and differential equations is a matter of a mathematical convenience or aesthetic taste. For empirical models, however, the choice of time scale can crucially affect estimation and verification. Preferably, it should be determined by the dynamic nature of the process under investigation. For example, in population dynamics, the period of reproduction of nonoverlapping generation insects can be used as the natural unit to construct a difference equation. More general systems that exhibit continuous motion with a natural or intrinsic period should be sampled at intervals that correspond with the intrinsic frequency. The resulting discrete time series can then be described by a difference equation. However, when the natural period of a process is not known, the choice of time unit is an open question. We cannot arbitrarily choose the time unit without theoretical analysis and empirical evidence. This would appear to be the viewpoint we should take in economics as noted by Koopmans. He suggested replacing the discrete-time stochastic model with a continuous-time stochastic model when the serial correlation is much longer than the time unit (Koopmans, 1950).

Figure 15.5 shows that the pattern of the phase portrait is sensitive to the time lag T for discrete mapping, but not sensitive for continuous-time ones. The latter changes its shape only with varying T (Figure 15.4). In either case, the time unit plays a critical role in data analysis.

To illustrate the problem, consider a discrete-time Henon model economy and assume the intrinsic unit is a month. Now look at the phase portrait for "quarterly" or "annual" samples. The phase portraits in Figure 15.5 show that the pattern of quarterly data is more complex than that of monthly data. It also illustrates that the image of the annual data appears like random noise except its square boundary. Fitting the Henon model to quarterly and annual data leads to complete failure. This example illustrates why, if the underlying economic dynamics are truly discrete and its intrinsic time unit is the order of day, or week, or month, the quarterly or monthly economic indicators are not capable of revealing the discrete nature of dynamic process.

Numerical experiments show that the autoregressive and moving average model

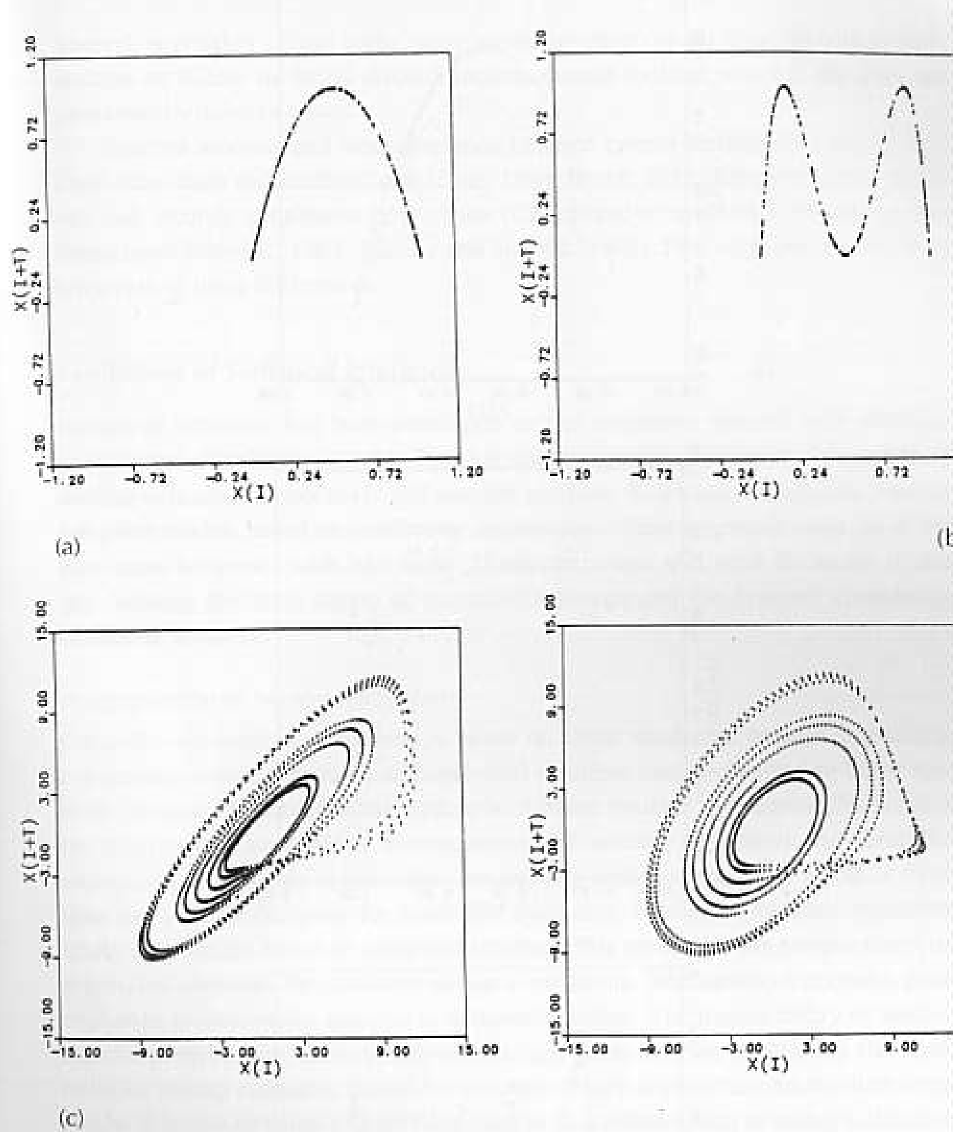


Figure 15.4 The two-dimensional phase portrait $X(I)$ vs $X(I+T)$ of deterministic chaos with varying time lag T. (a) Logistic model with $T = 1$, time interval $\Delta t = 1$, data size $N = 300$. (b) Logistic model with $T = 2$, $\Delta t = 1$, $N = 300$. (c) Rossler model with $T = 0.5$, time interval $\Delta t = 0.05$, and $N = 1000$. (d) Rossler model with $T = 1$, $\Delta t = 0.05$, $N =$ 1000.

(ARMA) can well represent data generated by discrete time chaos, such as the Henon and logistic models when the time intervals are the intrinsic ones but not when they are based on a sample at time intervals different than this. For continuous time models, like those of Lorenz (1963) or Mackey–Glass (Mackey and Glass, 1977), the ARMA model can fit the generated data only when the sampling time

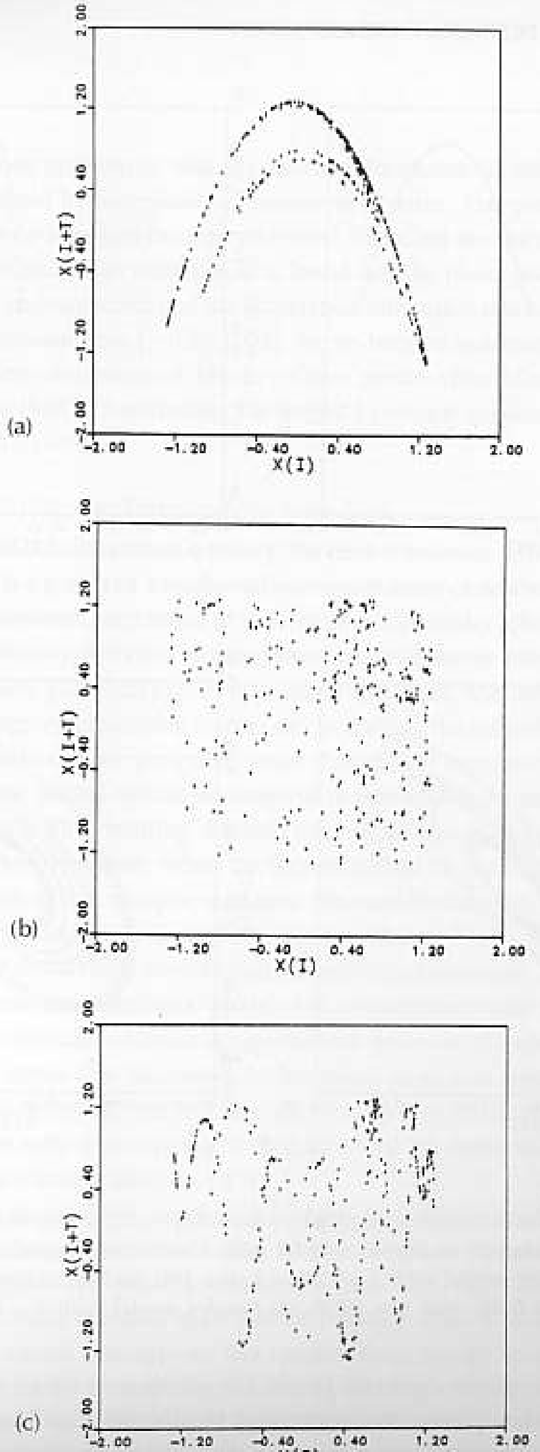


Figure 15.5 The phase portrait of the Henon economy observed from varying time interval Δt, $T = 1$, $N = 300$. (a) Original monthly data with $\Delta t = 1$. (b) Quarterly data with $\Delta t = 4$. (c) Annual data with $\Delta t = 12$.

interval is roughly of the same order as the average orbital time. In eithe success or failure in fitting ARMA models cannot indicate whether the d generated by noise or chaos.

Spectral analysis and autocovariance function cannot distinguish betwe crete-time chaos and random noise (Dale, 1984; Brock, 1986). However, thes ods can identify continuous-time chaos (Crutchfield et al., 1980; Grossm Sonneborn-Schmick, 1982; Nicolis and Nicolis, 1986). Few economic rese are aware of these differences.

Limitations of Statistical Inference

Statistical inference has been developed to test stochastic process with i independent distribution (i.i.d.). To what degree statistical inference is cap dealing with chaotic process is still an open question. Stochastic and chaotic are polar models based on conflicting assumptions. Most empirical cases li gray zone between chaos and noise. Econometricians will soon be aware gap between the static nature of statistical inference and the dynamic con of chaotic behavior.

Inseparability of Nonlinear Systems

Currently, econometric reasoning is based on linear stochastic models. Int components can be separated and analytical solutions can be obtained in lin tems, because the superposition principle of linear systems mathematically the theoretical framework of homogeneous and additive economies. Howe superposition principle is not valid for nonlinear systems, since the whole than the sum of the parts for nonlinear dynamics. Nonlinear dynamic e rarely have closed forms of analytical solution. This situation casts serious d regression exercises for nonlinear dynamic problems. Nonlinearity imposes challenge to time-series analysis in economic studies. The inseparability of ear components may frustrate econometricians when they are developing st tools for testing economic chaos. For example, Brock argues that chaotic tim can be detected by using a linear filter such as first differencing, or taking re a fitted ARMA model. He believes that the dimensionality of the original an tered time series should be the same (Brock, 1986). However, it is difficult the result of the differencing operation because it is sensitive to the time of differencing. The concept of fractal dimensionality comes from self-simi fractal structures (Hentschel and Procaccia, 1983). Brock did not discuss t of self-similarity when he tried to prove the residual test theorem.

The complexity of the problem can be seen from a special case of firs encing. Assume $\{X(t)\}$ denotes the continuous time series generated from a attractor, say, the Rossler attractor. The fractal dimension of Rossler attrac larger than two but less than three. A one-dimensional chaotic discrete-tin $\{u_n\}$ can be obtained from the Poincare section of $\{X(t)\}$ in a two-dim

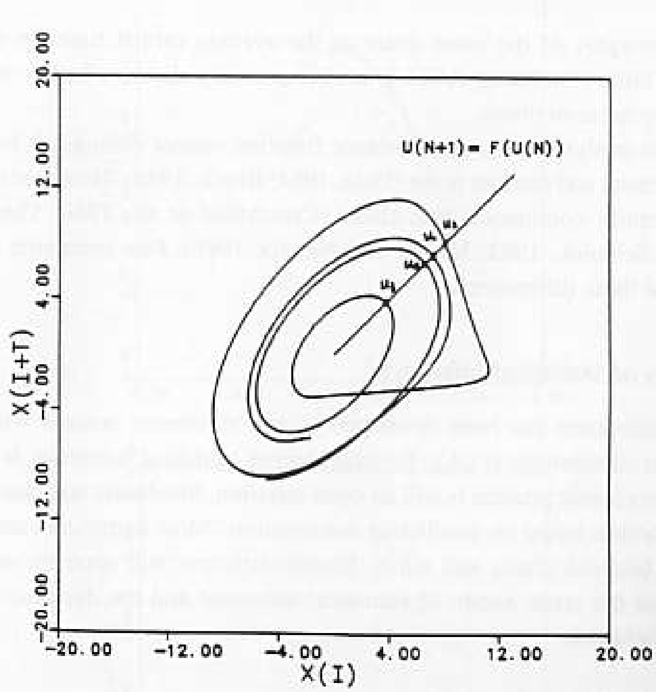


Figure 15.6 The relationship between continuous and discrete chaos. The discrete-time chaos $\{u_n\}$ is obtained from the one-dimensional Poincare section of continuous-time chaos $\{X(I)\}$ in a two-dimensional phase space. Here, $X(I)$ is generated from the Rossler model with $T = 1, \Delta t = 0.05, N = 500$. The time unit of one-dimensional discrete map $u_{n+1} = F(u_n)$ is the average orbital time $T_n = 1/f_n$ around the unstable equilibrium point of continuous-time chaos $\{X(I)\}$. The natural frequency f_n can be determined from the highest peak in the power spectrum.

phase space. Its time interval is equal to the averaging orbital (natural) time T_N of the attractor (see Figure 15.6). Therefore, the fractal dimension D' of $\{u_n\}$ must be less than 1.

For the differenced time series $\{\Delta u_n]\}$, the outcome is uncertain if the time interval for differencing is arbitrarily chosen. In practice, the differencing procedure in econometric modeling is a "whitening" process that cuts off the autocorrelation and increases the variance in observed time series. So far as we know, there is no theoretical argument and numerical evidence to show the invariance of dimensionality under a difference transformation.

Changing Strangeness under the Residual Test

It has been noted that correlation dimension is not invariant to a smooth coordinate transformation (Ott, Withers, and Yorke, 1984). The residual of a moving average process introduces random noise into the original data. This procedure may erase the fractal structure (Garcia-Pelayo and Schieve, 1991). For the autoregressive process, the situation becomes more subtle. The metric fractal dimension under a smooth

linear deterministic transformation is invariant, but most probability dimensions ar not.

To check the validity of the residual test, we tested the Henon attractor wit 5000 data points, which is good enough to uncover its dimensionality (Ramse and Yuan, 1989). We fit the ARMA(6,3) model and AR(6) model, respectivel to the Henon time series. The correlation dimension of ARMA(6,3) residuals i equal to the embedding dimension, which is the characteristic of random noise. Th correlation dimension of AR(6) residuals cannot be determined because no parall line can be identified from the GP plot. Probably, the AR(n) transformation change the probability density in phase space, and the definition of correlation dimensio is related to the square of probability density (Hentschel and Procaccia, 1983). residual test of the logistic time series has a similar result. For a continuous tim model, such as the Lorenz attractor, fitting it to the low-order ARMA model increasingly difficult, when autocorrelation is long and the time interval is sho compared with its natural orbital time. No clear-cut conclusion can be reached fro the residual tests in our numerical experiment.

A technical remark should be made here. It is known that only idealized mo els of pure random noise and well-behaved attractors have well-defined correlati dimension. This means that a time series may not have a well-defined correlation d mension. When no plateau region or no saturated dimension can be identified fro the Grassberger–Procaccia plots, the correlation dimension is not tractable. The fa value of the numerical mean in dimension calculations should not be readily a cepted until its Grassberger–Procaccia plot has been carefully examined.

The Pitfall of Linear Stochastic Filter in Detecting Deterministic Chaos

It is well known that any Gaussian or i.i.d. time series can be represented by infinite autoregressive process or moving average process (Granger and Newbo 1986). Some features observed in empirical tests, such as the long autocorrelatio in a deterministic time series, can be simulated by a finite-order stochastic proce either in linear or nonlinear form. But a stochastic model cannot simulate a se similar structure such as that of the Cantor set. Characterizing a strange attrac requires a spectrum with infinite dimensionality (Farmer, Ott, and Yorke, 1983).

The above discussion may help to solve the dispute raised by Ramsey, Saye and Rothman (1990). Conflicting results from nonlinear diagnostics and the resid test are reported in testing the monetary indexes (Chen, 1987b, 1988b; Barn and Chen 1988; Ramsey, Sayers, and Rothman, 1990). Ramsey and co-work duplicated our results from log-linear detrended data. However, there is no sign nonlinear structure in the residuals resulted from a double-sided moving aver filter. The reason for the absence of such a sign is that the symmetric, low-rej filter used by Ramsey and his colleagues did not wipe out high-frequency noise, improperly removed the low-frequency deterministic components. As we indica before, a continuous-time chaos can be considered as an imperfect periodic mot with low frequency and irregular amplitude (Chen, 1988b). In Ramsey's test, filtered time series did not even become stationary, which was required by attractor modeling. The low-reject filter made the variance of the filtered monetary index increase over time. The seemingly contradictory reports resulting from the residual tests are actually an aid in understanding the essential difference between linear stochastic deduction and nonlinear deterministic logic.

The Roots of Nonstationality and Nonnormality

Nonstationality and nonnormality are widely observed in economic time series because economies are open systems. It is a formidable task for econometricians to deal with these problems within the conventional framework of i.i.d. process. Deterministic approach and stochastic approach in theoretical economics represent conflicting ideas of endogenous and exogenous mechanisms of business fluctuations. However, the deterministic description and probabilistic description of dynamic process in theoretical physics are simply complementary tools in the unified dynamical framework. For example, chemical reactions can be described by (deterministic) differential equations or a master equation. The probability distribution function in master equation can be obtained by solving a (deterministic) partial differential equation. In the case of the Fokker–Planck equation, the peak of the distribution function or the mean value evolves along the path that can be represented by the trajectory of the corresponding deterministic equation. Therefore, these two approaches are equivalent when the distribution function is unimodal (Nicolis and Prigogine, 1977, 1989; Reichl, 1980). However, during bifurcation at the critical point of some control parameter, fluctuations will be so large that the mean value no longer represents the most-likely situation because the distribution function may become multihumped (Baras et al., 1983; Chen, 1987a). Actually, many statistical practitioners have already observed nonnormal, long-tail, and multimodal distribution in empirical studies.

The relationship between deterministic approach and probabilistic approach in nonlinear dynamic systems is illustrated in Figure 15.7.

Roughly speaking, between two bifurcation points, the dynamic process follows a deterministic path, which can be described by averaging when the process has a unimodal distribution. Statistic inference or i.i.d. process can be approximately applied only in this situation. The bifurcation model is quite useful in understanding noncontinuity, nonstationality, and nonnormality in real economies when econometricians are confounded by the multiple phase character of economic evolution (Day and Walter 1989). Changing economies can be one of the major obstacles in detecting economic chaos.

Some Conclusion about Numerical Algorithms

Practically, we have only some clue of low-dimensional attractors with finite data sets. There is no way to identify deterministic chaos with certainty. At present, with data only in the hundreds, the discovery of economic strange attractors whose dimensionality is higher than 3 is unlikely.

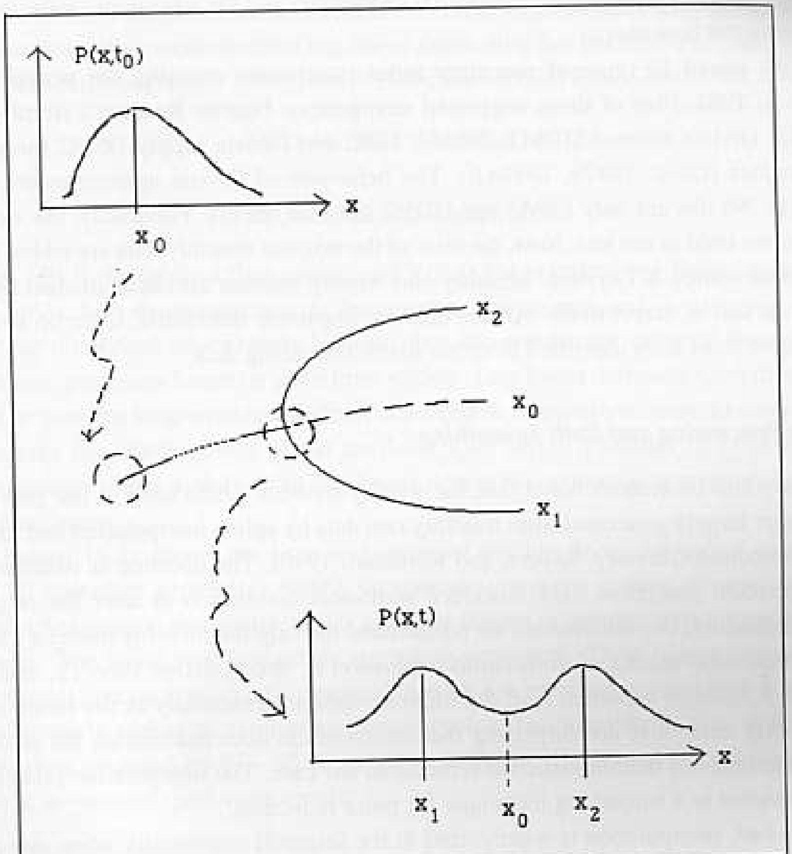


Figure 15.7 The relationship between the probability distribution function of a stochastic equation and the corresponding solution of the deterministic equation.

We can only speculate why we were unable to identify correlation dimensic for other types of economic time series, such as GNP, IPP, and the Dow–Jones ind in our numerical tests. Either their dimensions are too high, or their noise levels too large, or they do not in fact reflect intrinsic characteristics of complex behavi or changing economies caused by series of bifurcations. Current observational sc and analytical technique are not capable of solving these problems.

Testing Economic Chaos in Monetary Aggregates

With the pitfalls well in mind, I shall briefly reconsider my work on monetary ag gates to illustrate techniques for detecting chaos. The Federal Reserve's mone indexes include M1, M2, M3, and L. These are simple-sum aggregate indexes (noted by SS hereafter). There are also parallel Divisia theoretical indexes suc

Divisia demand indexes (denoted by DD hereafter) and Divisia supply indexes (denoted by DS hereafter).

We tested 12 types of monetary index time series covering the period from 1969 to 1984. Five of them suggested strangeness: Federal Reserve's simple-sum SSM2, Divisia demand DDM2, DDM3, DDL and Divisia supply DSM2 monetary aggregates (Chen, 1987b, 1988a,b). The behaviors of Divisia aggregates are very similar. We discuss only SSM2 and DDM2 here for brevity. Previously, the weekly data were used in our test. Now, the tests of the original monthly data are added here. Our data source is Fayyard. Monthly and weekly indexes are distinguished by the letter m and w, respectively. All the data are log-linear detrended, since no strange attractors have been identified from the first differencing data.

Data Processing and Path Smoothing

Ramsey and co-authors noted that the weekly monetary data used in our previous test were largely generated from monthly raw data by spline interpolation and model reconstruction (Ramsey, Sayers, and Rothman, 1990). The question is whether the interpolation procedure may introduce additional correlation or alter the original dimensionality. For this reason, we reexamined the original monthly monetary data. The numerical results of correlation dimension n, decorrelation time T_d, and the largest Lyapunov exponent λ of the monthly data are essentially in the same order as weekly data. It is not surprising that interpolation does not change the primary characteristics of deterministic movements in our case. The interpolation procedure is equivalent to a smoothing technique for noise reduction.

In fact, interpolation is widely used in the scientific community when raw data are incomplete (Charney, Halem, and Jastrow, 1969; Tribbia and Anthes, 1987). Interpolation and smoothing were also used in testing chaos from climate, ecological, and epidemic time series (Grassberger, 1986; Schaffer, 1984; Schaffer and Kot, 1985).

Detrending Methods and Attractor Models

Testing economic aggregate time series for chaos or randomness is a formidable task. The intrusion of growth trends raises a critical problem of how to characterize a growing economy by means of mathematical attractors. Various methods of detrending have been used in econometrics. We are interested in their theoretical implication: the choice of reference system in observing economic behavior. We attempted to explore this problem through numerical experiments on empirical data.

For example, the percentage rate of change and its equivalent form, the logarithmic first differences, are widely used in fitting stochastic econometric models (Osborne, 1959; Friedman, 1969). It can be defined as follows:

$$Z(t) = \ln S(t+1) - \ln S(t) = \ln \{S(t+1)/S(t)\} \quad (15.4)$$

where $S(t)$ is the original time series, and $Z(t)$ is the logarithmatic first differer

An alternative method called log-linear detrending has been used in chaos els (Dana and Malgrange, 1984; Brock, 1986; Barnett and Chen, 1988). We ha

$$X(t) = \ln S(t) - (k_0 + k_1 t)$$

or

$$S(t) = S_0 \exp(k_1 t) \exp[X(t)$$

where $S(t)$ is the original time series, and $X(t)$ is the resulting log-linear detr time series, k_0 is the intersection, k_1 the constant growth rate, and $S_0 = \exp(k_0)$

Our numerical experiments indicate that the percentage rates of chan whitening processes based on short time scaling. Log-linear detrending, on the hand, retains the long-term correlations in economic fluctuations, since its time represents the whole period of the available time series. Findings of evider deterministic chaos mainly from log-linear detrended economic aggregates l this conclusion.

Figure 15.8a shows the time sequences of the log-linear detrended (de by LD) monetary aggregates SSM2. Its almost symmetric pattern of nearly length of expansion and contraction is a typical feature of growth cycles in eco systems. The usual business cycles are not symmetrical. Their longer expa and shorter contractions can be obtained in such a way, when a trend with co growth rate is adding to symmetric growth cycles. The logarithmic first-diff time series (denoted by FD) SSM2 is given in Figure 15.8b as a comparison latter is asymmetric and more erratic.

Empirical Evidence of Deterministic and Stochastic Processes

Based on the phase portrait and autocorrelation analysis, we can qualitatively guish a stochastic process from a deterministic one. Figure 15.9a presents the portrait of detrended monetary aggregates LD SSM2. It rotates clockwise li spiral chaos in Figure 15.2c. The complex pattern is a potential indication of r ear deterministic movements and eliminates the possibilities of white noise or periodic motions. To compare with a series that appears like white noise, the portrait of IBM daily stock returns is shown in Figure 15.9b. The autocorre of the detrended time series are shown in Figure 15.10. Readers may compare with the autocorrelations in Figure 15.3c.

If we approximate the fundamental period T_1 by four times the decorr time T_d, as in the case of periodic motion, then, T_1 is about 4.7 years for LD This result is very close to the common experience of business cycles. We will to this point later.

The Numerical Maximum Lyapunov Exponent and Autocorrelations

Let us now consider the tests using the Lyapunov experiments and autocorrel In theory, the choice of evolution time EVOLV, embedding dimension m an

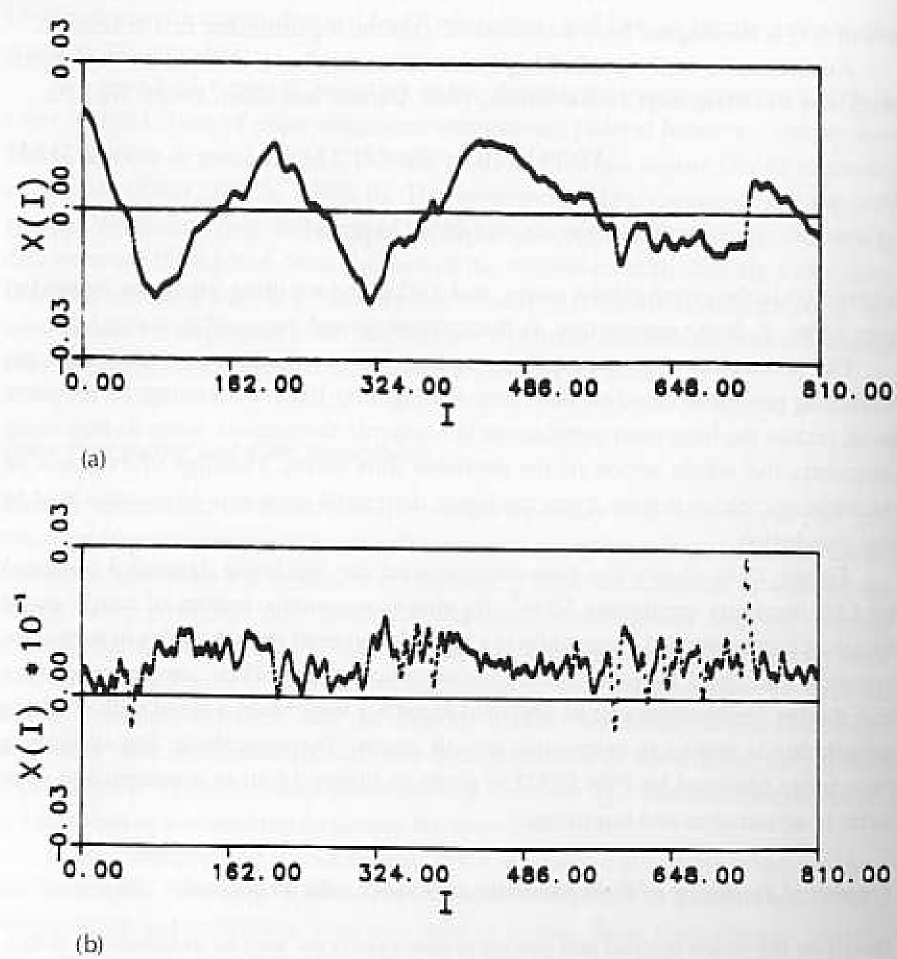


Figure 15.8 Comparison of the detrended weekly time series SSM2w. (a) Symmetric LD SSM2w: the log-linear detrended SSM2w with a natural growth rate of 4% per year. (b) Asymmetric FD SSM2w: the logarithmic first differences of SSM2.

delay T, has no relevance to the maximum Lyapunov exponent. In practice, the value of the Lyapunov exponent does relate to the numerical parameters. The range of evolution time EVOLV must be chosen by numerical experiments. The positive maximum Lyapunov exponents of the investigated monetary aggregates are stable over some region in evolution time. The numerical Lyapunov exponent is less sensitive to the choice of embedding dimension m. In our tests, we fixed m at 5 and time delay T at 5 weeks based on the numerical experiments. For example, the stable region of EVOLV is 45–105 weeks for SSM2 and 45–150 weeks for DDM2. Their average maximum Lyapunov exponents over this region are 0.0135 and 0.0184 (bit per week), respectively.

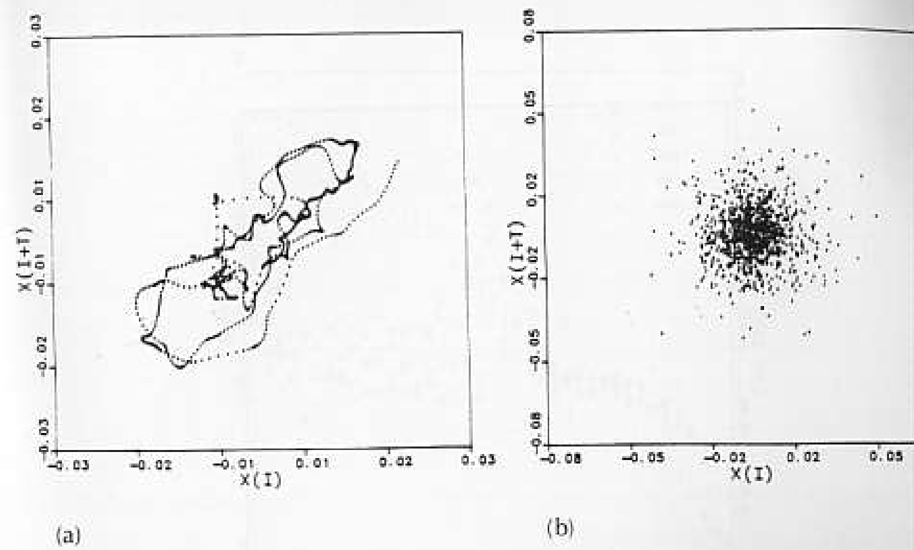


Figure 15.9 Comparison of the phase portraits of empirical time series. Time de
$T = 20$. (a) LD SSM2w weekly time series. $N = 807$ points. (b) IBM daily common st
returns. $N = 1000$ points, beginning on July 2, 1962.

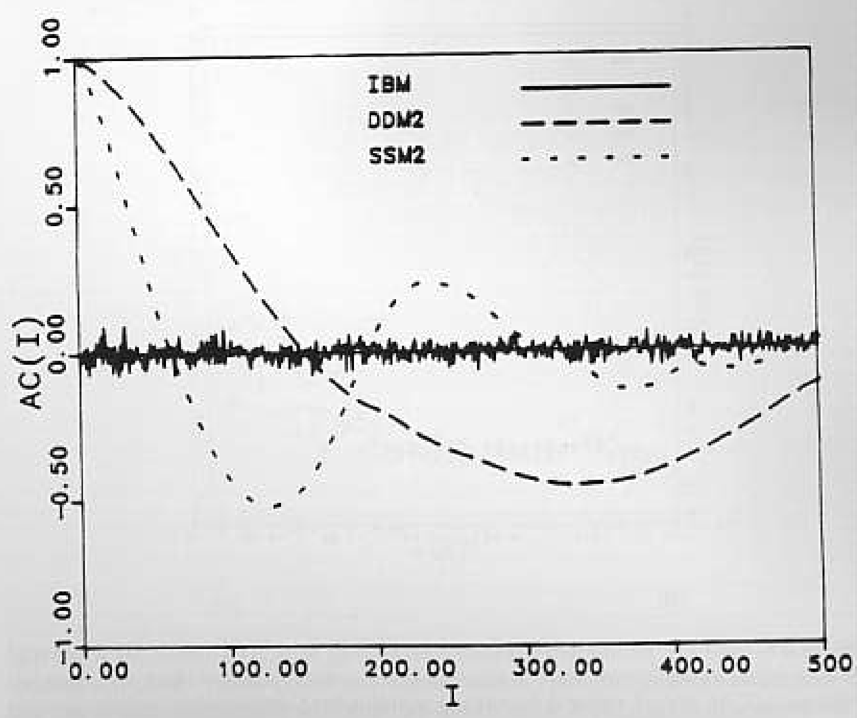


Figure 15.10 Comparison of autocorrelation functions; $AC(I)$ plotted against I. T
are three time series: LD SSM2w and LD DDM2w weekly data, and IBM daily s
returns, each in the same time units as in Figure 15.9. $N = 807$.

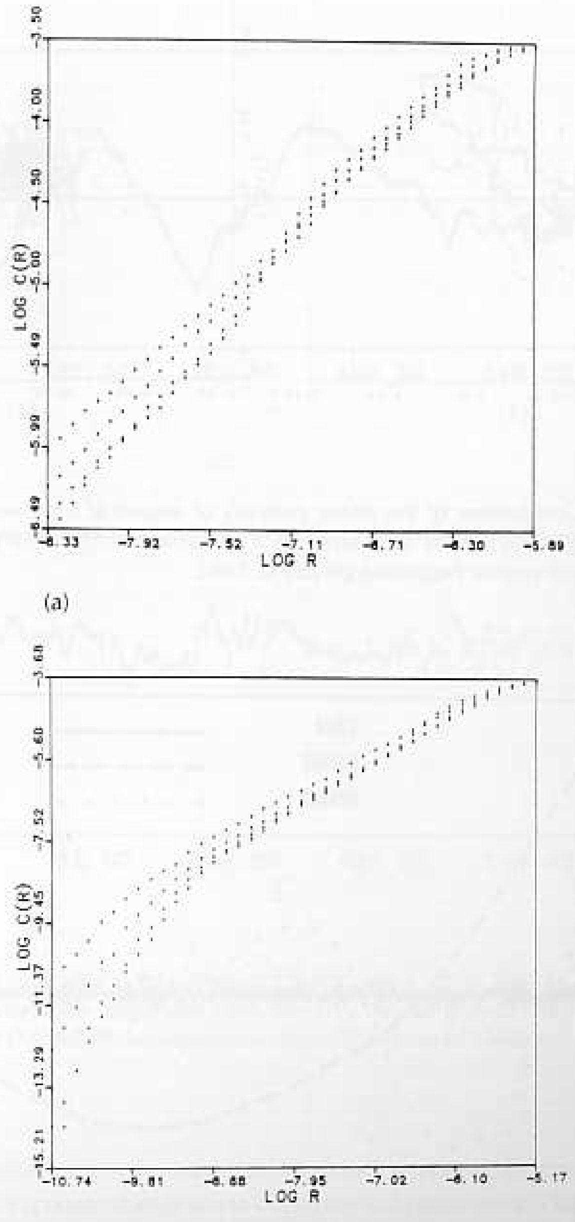


Figure 15.11 The GP (Grassberger–Procaccia) plot of $\ln_2 C_m(R)$ versus $\ln_2 R$ for log-linear detrended monthly monetary indexes beginning from January 1969. The embedding dimension, $m = 1, \ldots, 5$, is taken as a parameter. The plots rotate downward and to the right as m increases. The time lag $T = 10(m)$. (a) SSM2m, $N = 195$ observations. (b) DDM2m, $N = 195$ observations.

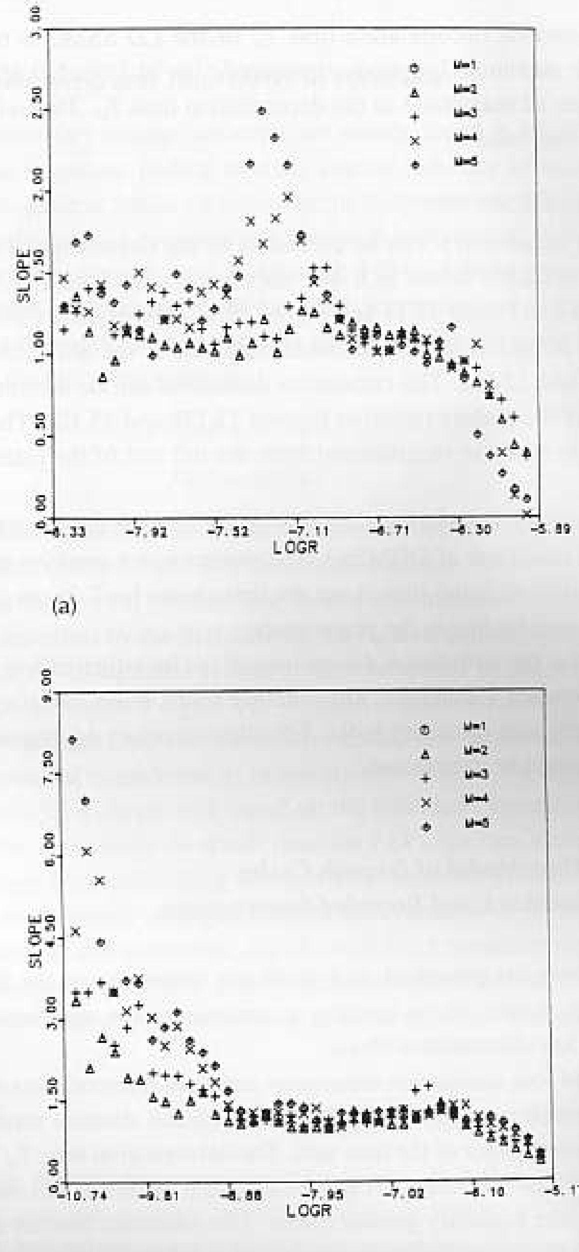


Figure 15.12 The GP Plot of SLOPE_m versus $\ln_2 R$ for the same detrended monet indexes. The correlation dimension n is estimated from the saturated slope in plate regime, which corresponds to the linear regime of lines in Figure 15.11. (a) SSM2 $N = 195$ observations. (b) DDM2m, $N = 195$ observations.

The characteristic decorrelation time T_d of the LD SSM2 is 61 weeks. The reciprocal of the maximum Lyapunov exponent $\lambda^{-1}(=74.1)$ for LD SSM2 is roughly of the same order of magnitude as the decorrelation time T_d. This relation does not hold for pure white noise.

The Correlation Dimension

The correlation dimension ν can be estimated by the Grassberger–Procaccia algorithm. Plots of ln $C_m(R)$ versus ln R and slope versus log R for LD SSM2 and LD DDM2 are shown in Figure 15.11 and Figure 15.12. The existence of linear regions of intermediate R, which reflect the fractal structure of the attractors, is shown in Figures 15.11a and 15.12a. The correlation dimension can be determined from the up-bond slope of the plateau region in Figures 15.11b and 15.12b. The level of uniform noise in the data can be estimated from the left end of the plateau when R is small.

We calculated the correlation dimension with the time lag T varying from 4 to 38 (the decorrelation time of DDM2m). The pattern is not sensitive to changing T. The first zero-autocorrelation time is not the best choice for T in our cases, because a large T may cause folding in the phase space.

We found that the correlation dimensions of the investigated five monetary aggregates were between 1.3 and 1.5. They include four Divisia monetary indexes and one official simple-sum monetary index. For other monetary aggregates, no correlation dimension could be determined.

A Continuous Time Model of Growth Cycles with Delayed Feedback and Bounded Expectations

Given the evidence just presented, and given our comments on the desirability of a continuous-time model, let us consider a continuous-time representation of economic data with low-dimensional chaos.

The observed low correlation dimension and long decorrelation–time set constraint on the modeling of growth cycles. For a typical discrete model, T_d is approximately the same order of the time unit. The decorrelation time T_d for monetary attractors is more than 60 weeks. A continuous time model would seem to be appropriate to describe monetary growth cycles. The minimum number of degrees of freedom required for chaotic behavior in autonomous differential equations is 3. The low dimensionality of monetary attractors leads to the assumption that the monetary deviations are separable from other macroeconomic movements. The background of growth cycles can be approximately represented by a constant exponential growth trend, or the so-called natural growth rate.

After comparing the correlation dimension and the phase portraits of the data and alternative models, a differential-delay equation suggests itself as a good candidate. For simplicity, we consider only one variable here.

Deviations from Trend and Time Delay in Feedback

The apparent monetary strange attractors are mainly found in log-linear detrend data. This is an important finding to study control behavior in monetary policy assume that the general trends of economic development are perceived by peo in economic activities as a common psychological reference or as the anchor in serving and reacting (Tversky and Kahneman, 1974). Administrative activities basically reactions to deviations from the trend. Accordingly I choose the deviati from the "natural growth rate" as the main variable in the dynamic model of mo tary growth in the following equation:

$$dX(t)/dt = aX(t) + F[X(t-\tau)] \quad (15$$

$$F(X) = XG(X) \quad (15$$

where X is the deviation from the trend, τ is the time delay, a is the expansion spe F is the feedback function, and G is the control function. There are two compet mechanisms in the growth system. The first is the immediate response to mar demand. It is described by the first term on the right of equation (15.1). The seco term represents the endogenous system control described by the feedback funct F. This consists of feedback signal $X(t-\tau)$ and control function G. The time de τ exists in the feedback loop because of information and regulation lags.

There are several considerations in specifying F and G. We argue that the m etary policy follows a simple rule based on the bounded expectations of monet movements. We assume the feedback function $F(X)$ has two extrema at $\pm X_m$ the control-target floor and ceiling as argued by Solomon (1981). To describe overshooting in economic management and the symmetry in growth cycles, G should be nonlinear and symmetric, $G(-X) = G(X)$. A simple exponential funct describes the assumed nonlinear control function with flexible floor and ceiling control behavior is similar to that driving in a freeway with lower and upper sp limits.

$$G(X) = -b\ \exp(-X^2/\sigma^2) \quad (1$$

where b is the control parameter, the minus sign of b is associated with n tive feedback, σ is the scaling parameter, and the extremas of $F(X)$ are locate $X_m = \pm\sigma/\sqrt{2}$. Substituting equation (15.9) into equations (15.7) and (15.8) gives following differential-delay equation:

$$dX(t)/dt = aX(t) - bX(t-\tau)\ \exp[-X(t-\tau)^2/\sigma^2] \quad (15$$

We may change the scale by $X = X'\sigma$ and $t = t'\tau$, then drop the prime for co nience:

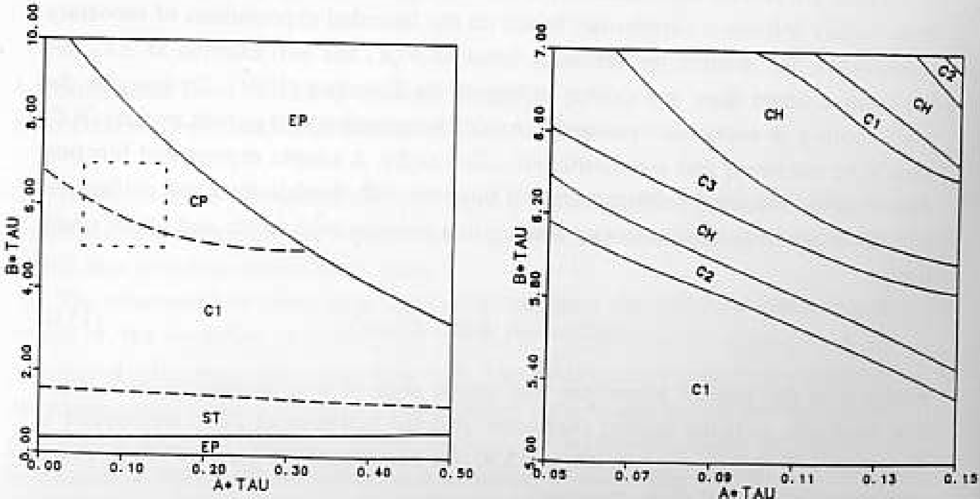
$$dX(t)/dt = a\tau X(t) - b\tau X(t-1)\exp[-X(t-1)^2] \quad (15$$

What Can We Learn from the Model

No empirical phenomenon can be understood without theoretical reasoning. Only some empirical tests offer supportive arguments for theoretical judgment. The choice of chaotic model over stochastic model largely depends on whether we can obtain more information based on the same set of empirical data. Our answer is yes. The mechanism of intrinsic instability and the pattern of irregularity illustrated in nonlinear chaotic models are entirely foreign to linear stochastic models.

Phase Transition and Pattern Stability

Figures 15.13a–b displays qualitatively the phase diagram of equation (15.11) in the parameter space. The broad diversity of dynamic behavior includes steady-state ST, limit cycle or periodic motion C1, and the explosive solution EP. The complex regime CP includes multiperiodic (C1, C2, C3) and chaotic regime CH. When parameter values change within each region, the dynamic behavior is pattern-stable, because the dynamic mode occupies a finite area in the parameter space. The phase transition occurs when parameters cross the boundary between different phases. It is observable when the wave pattern changes.



(a) (b)

Figure 15.13 The phase diagram of numerical solutions of equation (15.11) in parameter space $a\tau$ and $b\tau$. The dashed area in (a) is enlarged in (b). Here, EP, ST, and CP represent explosive regime, steady state (after damped oscillation), and complex regime, respectively. CH is chaotic regime. C1, C2, and C3 are periodic patterns, whose longer wave consists of one, two, or three shorter waves in turn.

Long Wave and Short Cycles

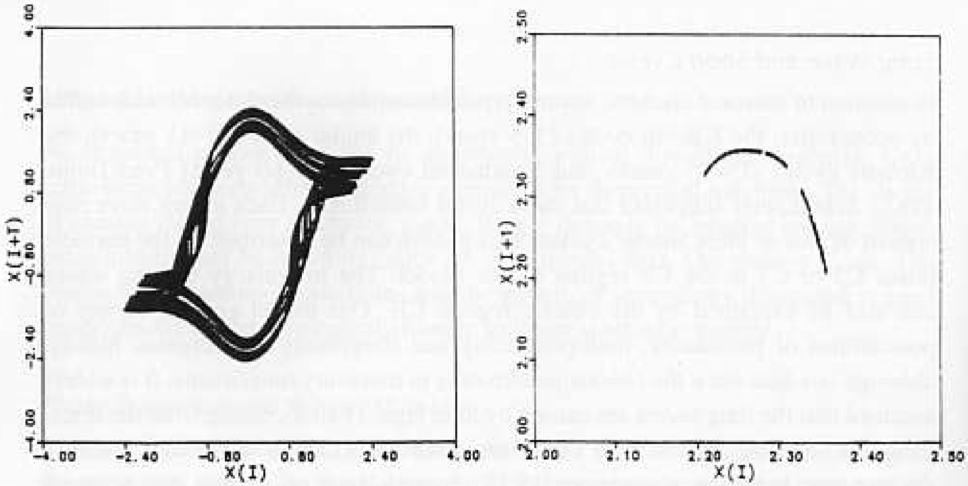
In addition to seasonal changes, several types of business cycles have been identifie by economists: the Kitchin cycles (3–5 years), the Juglar cycles (7–11 years), th Kuznets cycles (15–25 years), and Kondratieff cycles (45–60 years) (Van Duij 1983). Schumpeter suggested that these cycles were linked. Each longer wave ma consist of two or three shorter cycles. This picture can be described by the period phase C2 or C3 in the CP regime of our model. The irregularity in long wav can also be explained by the chaotic regime CH. Our model gives a variety possibilities of periodicity, multiperiodicity and irregularity in economic histor although our data show the chaotic pattern only in monetary movements. It is wide assumed that the long waves are caused by long lags, a belief coming from the line paradigm (Rostow, 1980). This condition is not necessary in our model, becau the dynamic behavior of equation (15.13) depends both on $a\tau$ and $b\tau$. A stro overshooting plus a short time delay has the same effect as a weak control pl a long time delay, a point also made by Sterman (1985). This model is so simp and general, it could have applications beyond the monetary system in the mark economy we discussed here. For example, the growth cycles and long waves caus by overshooting and time delay may also happen in centrally planned economies.

Simulating Empirical Cycles and Forecasting Basic Trends

In comparing model-generated patterns with empirical data, we may confine experiments to certain regions of the parameter space. For example, we can estim the average period T from 4 times the decorrelation time T_d. The time delay τ monetary control due to regulation lag and information lag is between 20 and weeks (Gordon, 1978). If we estimate the time delay τ to be 39 weeks, we simulate LD SSM2 time series by the solution by setting $\tau = 39, a = 0.00256,$ 0.154, and $\sigma = 0.0125$. The model results match well the average amplitude decorrelation time T_d, positive maximum Lyapunov exponent λ, and correlat dimension D of the empirical time series.

We tested the theoretical models with power spectra and autocorrelation an ysis. The approximated period T of the chaotic solution can be estimated from measured by 3–5 cycles. It is close to the fundamental period $T_1(=f_1^{-1})$. The f damental frequency can be determined by power spectra. The error can be less t 3% when observation period covers 100 cycles. For LD SSM2 time series, the ference of T_d measured between 10 and 15 years is less than 5%. We can ob valuable information about the fundamental period T_1 without knowing the e parameters of the deterministic model.

The small data sets cause the estimation of the correlation dimension to be ased downward (Ramsey and Yuan, 1989). In our theoretical simulation of mone cycles, the numerical result of the correlation dimension is 1.7 calculated with data points and 2.08 with 16,384 points for the same model. The error is 18 dimension estimation of the growth-cycle model. The results of monthly mone



(a) (b)

Figure 15.14 The two-dimensional phase portrait and the maximum map of theoretical monetary strange attractor (Chen, 1987b). The scale is arbitrary. (a) The two-dimensional phase portrait of theoretical monetary strange attractor, which rotates clockwise, $N = 25,000$, $T = 1$, $\Delta t = 0.01$. (b) The maximum map obtained from the one-dimensional Poincare section in two-dimensional phase space. The time unit is the averaging orbital time T_n.

data are still within the margin of numerical reliability because of their low dimensionality (Chen, 1988b). Certainly, the numerical estimation of the correlation dimension for monetary indexes is only suggestive since the amount of data is small. Further empirical observations are needed to provide better evidence of monetary chaos.

The phase portrait and maximum map of a theoretical model of monetary chaos are demonstrated in Figure 15.14. The theoretical monetary strange attractor rotates clockwise (Figure 15.14a), which mimics the movement of the monetary growth cycle in Figure 15.9a. The maximum map shows the qualitative picture of discrete-time chaos typically created by the Poincare section (Figure 15.14b). A brief summery of both empirical and simulating results is given in Table 15.1. The time unit of weekly data is converted to monthly for comparison. Here, the time scales are 1 year = 12 months = 52 weeks, and 1 month = 4.3 weeks. SSM2t is the simulating time series generated by the theoretical monetary growth model and its correlation dimension n is calculated with 1000 and 16,384 data points, respectively (Chen, 1988b).

Implications for Forecasting and Control Policy

The predictive power of a chaotic economic time path is limited by the magnitude of the maximum Lyapunov exponent. Nevertheless, we may potentially recover more information from chaotic motion than from randomly generated movements. We know that a long-term prediction of the chaotic orbit is impossible from the view

Table 15.1 Empirical and Theoretical Evidence of Monetary Chaos

Name	N (obs)	$\lambda(/m)$	$\lambda^{-1}(m)$	$T_d(m)$	ν
SSM2m	195	0.0242/m	41.3m	14m	1.3
DDM2m	195	0.0489/m	20.4m	38m	1.3
DDM3m	195	0.0218/m	45.9m	37m	1.3
DDLm	192	0.0397/m	25.2m	35m	1.3
SSM2w	807	0.0581/m	17.2m	14.2m	1.5
DDM2w	807	0.0791/m	12.6m	34.9m	1.4
DDM3w	807	0.0774/m	12.9m	34.2m	1.5
DDLw	798	0.0525/m	19.1m	32.3m	1.5
DSM2w	798	0.0585/m	17.1m	32.6m	1.3
SSM2t	1000w	0.0688/m (16384)	14.5m	14.2m	1.7 (2.08)

of nonlinear dynamics. A medium-term prediction of approximate period T ca be made, if we identify strange attractors from the time series.

Let us discuss the meaning of the control parameters in equation (15.13). $b = 0$, the monetary deviation from the natural rate will grow at a speed $e^{\dots}$ define a characteristic doubling time t_a, which measures the time needed to c the autonomous monetary expansion $X(t)$ without control. Similarly, we ca fine a characteristic half time t_b, which measures the time needed to redu money supply to half its level, when $a = 0$ and $X(t - \tau)$ reaches the control $X_m = \sigma / \sqrt{2} = 1.4\%$ per year. The same is true for the contraction movements the feedback function $G(X)$ is symmetric. Here t_a is 5.2 year and t_b is 7.4 we SSM2 in our simulation. We see that even modest time delay and overshootin generate cycles and chaos.

For policy considerations, the phase diagram of the model suggests th steady state in money supply can be achieved by carefully adjusting the c parameter b or the time delay τ (see Figure 15.13). For example, we car (=0.1) and τ(= 39 weeks) and set b in stable regime ($0.41 < b < 1.51$ or 29.5 $< t_b <$ 108.7 weeks); we may also fix a(= 0.1) and b(= 6.0) but choose τ regime (14.4 minutes $< \tau <$ 1.3 day). Obviously, reducing overshooting is easier than cutting time delay in monetary control. These figures give a qual picture of monetary target policy (Chen, 1988b).

Linear Approximations of a Nonlinear Model

Let us study the relationship between the nonlinear dynamic model (15.10) linear approximation under some simplifying conditions.

A different equation can be obtained as an approximation of a no difference-differential equation (15.10) when the time unit is chosen to be th delay $\tau(\tau = 1)$ and $X(t)$ is much less than the control target σ, and b is less th we have

$$dX(t)/dt = X(n+1) - X(n) = aX(n) - bX(n-1)\ \exp(-X(n-1)^2/\sigma^2) \quad (15.12)$$

$$= aX(n) - bX(n-1) - bX(n-1)^3/\sigma^2 \quad (15.13)$$

or

$$X(n+1) = c_1X(n) + c_2X(n-1) + \omega X(n-1)^3 \quad (15.14)$$

where $c_1 = (1+a), c_2 = -b, \omega = b/\sigma^2$. Equation (15.14) looks like an AR(2) process when the nonlinear term $\omega X(n-1)^3$ is ignored and replaced by some noisy residual term. Although AR(2) approximation may be very useful and convenient in econometric analysis, its drawbacks and limitations are also significant. First, the sampling time unit should be the time delay that is between 20 and 56 weeks, equivalent to quarterly or annual data (Gordon, 1978). Second, the AR(2) model is misleading in its stochastic nature, because the residual is generated by the nonlinear term with long-term correlations, not random noise without correlations.

We may also have a differential version of equation (15.13) when the time delay is ignored,

$$dX(t)/dt = -aX(t)g\{\exp[-X(t)^2/\sigma^2] - 1\} \quad (15.15)$$

where $g = b/a >> 1$, so the equation has a fixed point solution. Friedman believes that the natural rate in economies can be achieved in so-called long-run equilibrium (Friedman, 1969). In our case, constant growth rate can be realized when the time delay in control process is zero. Obviously, this is an idealistic case but unrealistic situation. The concept of long-run equilibrium in static analysis can be considered as the fixed point solution in nonlinear dynamic systems. Although steady state is hard to achieve due to time delay and overreaction in human behavior, equilibrium or steady state can still serve a reference regime for control target.

This example demonstrates the close connection between nonlinear economic dynamics and linear dynamics. Static equilibrium analysis could be integrated in the generalized framework of disequilibrium dynamics.

Brief Summary and Future Directions

So far we have little evidence of economic chaos from empirical data. However, theoretical powers of modeling complex behavior and mathematical generality of nonlinearity strongly support the development of chaotic economic dynamics. The prerequisite for this advancement is its tremendous demand of empirical information and computational power to handle nonlinear problems.

The question is how to extend our scope of economic analysis and advance our study of economic chaos. Four directions should be explored in the near future.

- Rethinking the operational framework of chaos theory for empirical studies.

The standard definition of deterministic chaos is based on the positive Lya
exponent. Fractal dimension is also important in characterizing the strange att
(Hao, 1990). These criteria are useful in studies of theoretical models and con
experiments, but very restricted in empirical analysis. We should develop opera
guidelines for choosing chaotic or stochastic approaches in empirical analysis.

Prigogine pointed out that deterministic chaos is only a partial feature of
plex systems. Long-range correlation is the fundamental character of compl
namics (Nicolis and Prigogine, 1989). This definition of chaotic process ma
econometricians in understanding chaos and noise, since econometric anal
based on stochastic process with short correlations.

In my experience in analyzing large numbers of economic time series, ta
cases of economic chaos are rare, but long-term correlations appearing in em
data are abundant. The real problem is always more complicated than theo
models. It is true both for natural sciences as well as social sciences. For
ple, discovering the beautiful structure of hydrogen spectra is a rare case e
physics. However, the discontinuity of frequency distribution widely observed
tical spectra strongly support the quantum theory.

No empirical observation can be done without theoretical reasoning, whe
works in an explicit or implicit way. The difference lies deeply in theoretic fo
tion. Some econometricians use a whitening technique such as multiple differe
to eliminate correlations and justify stochastic models. We try long-term detre
methods to extract correlation signals and recover deterministic mechanisms.

- Reexamining the theoretical foundation of econometrics.

Economists often compare economy with weather (Goodwin, 1990). Alt
the irregularities of their behavior are very similar, their theoretical perspe
are just the opposite. The failure of econometric forecasting based on a stoc
approach (Dominguez, Fair, and Shapiro, 1988; Wallis, 1989) and the succ
weather forecasting based on a deterministic approach (Tribbia and Anthes,
dramatize the difference in their methodological foundation.

- Expanding the empirical base of economic studies.

Genuine economic dynamics cannot be discovered by curve fitting in
of statistical techniques. The current controversy of chaos versus noise can
completely settled by numerical tests based on limited contaminated data.

Consider the oldest problem of planet motion in astronomy. The irregula
paths of planet motion are easily seen in short-time observations. Linear sto
models may fit the data and give a good explanation of the short history. Ho
cumulative observations reveal regularity in recurrent patterns. Although arit
rules in calendar calculation can be established from empirical data, theoreti
derstanding went a long way from Copernicus's idea of heliocentric referen
tem, to Kepler's law of planet motion, and Newton's law of classical mechani

The improvement in weather forecasting has been achieved by expandi
data base through global weather-station networks and satellite-surveillance

niques. Increasing computer power also facilitates increasing precision of nonlinear models and weather forecasting. However, current resource constraints in empirical economic research result in an economic profession basically confined in thought experiments and linear models because of the lack of empirical data and computing power in the information age. Long-term investment in "economic weather-station network" and research efforts in complex economic dynamics is essential for advancing empirical economic science.

Exploring economic chaos opens new ways to understand human behavior and social evolution. The interdisciplinary character in developing evolutionary dynamics and nonlinear economics has not only changed the way we think, but also the institution in which we organize economic research.

Appendix A. A Direct Test for Determinism in Monetary Time Series

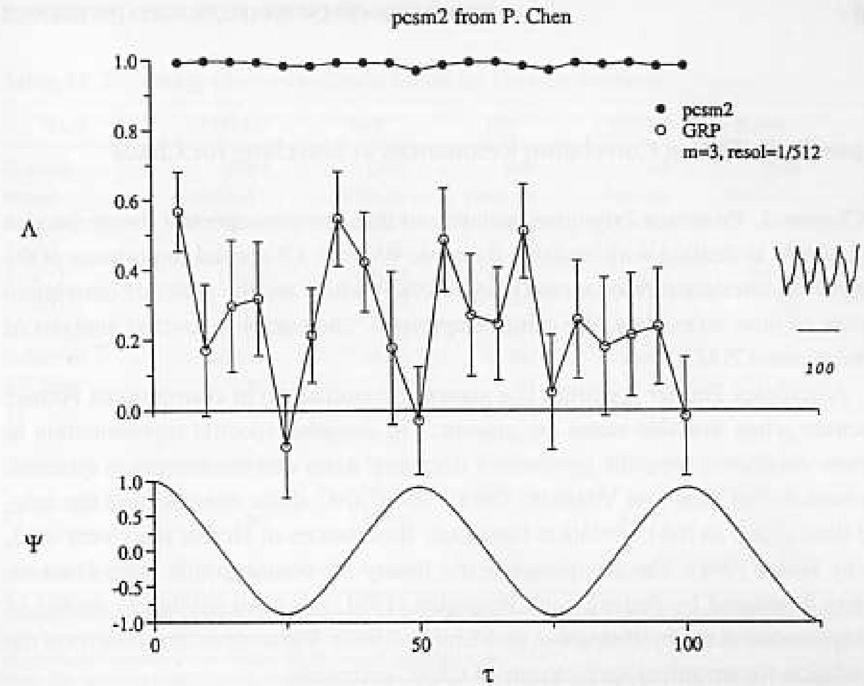
A new algorithm of direct testing determinism has been developed by Daniel Kaplan and Leon Glass (1992a,b) at McGill University. It turns out to be a useful tool to distinguish between determinism and randomness. Their idea is simple.

At each state a deterministic dynamic flow has only one direction, while a stochastic system has more than one possible value. By calculating local coarse-grained flow averages and statistics Λ, one may have a better chance of dealing with noisy and short time series than calculating Lyapunov exponents and correlation dimensions.

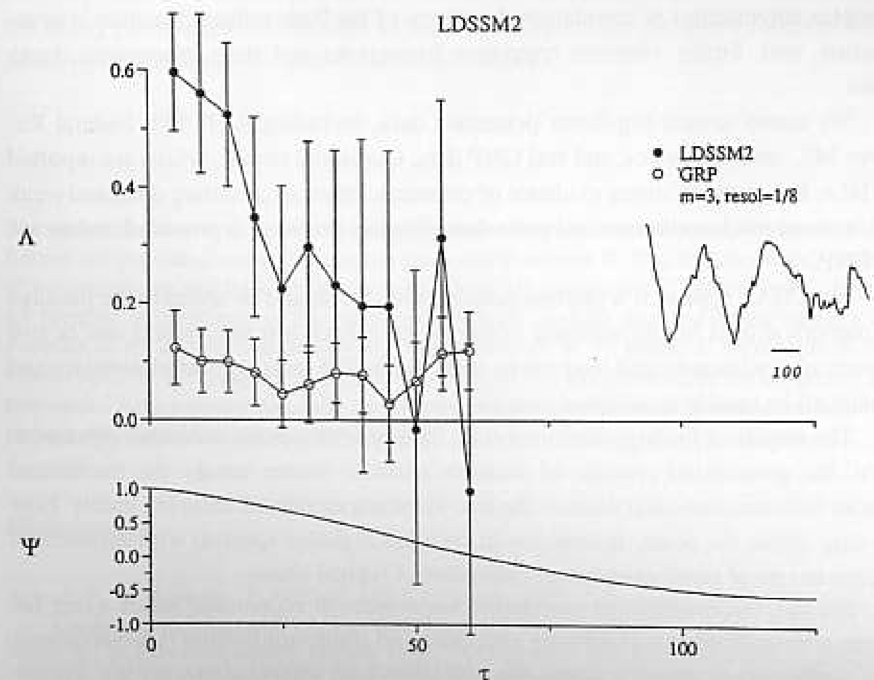
We sent two time series of 807 points in length, DLSM2 and PCSM2, to Kaplan. PCSM2 is a simulated time series generated by equation (15.11). DLSM2 is the log-linear detrended SSM2 weekly time series in Figure 15.9a. Kaplan finds clear conclusions for both time series. We now provide further evidence of deterministic monetary chaos, courtesy of Kaplan (private communication, April 9, 1992).

PCSM2 easily passes the test of determinism as shown in Figure 15.15a: the lambda-bar is close to 1 for the deterministic signal, and the contrasting Gaussian random process with the same autocorrelation function Ψ is much less and near zero, as is expected from the theory.

DLSM2 in Figure 15.15b shows that the signal is more deterministic than a Gaussian random noise. Its low resolution is due to short time series and high level of noise. It would be helpful for economic studies to have a longer time series of empirical data in the future.



(a)



(b)

Figure 15.15 Kaplan-Glass plots for testing determinism. Λ vs. τ for tested time series and a Gaussian random process (GRP) with identical autocorrelation $\Psi(\tau)$ are marked by open symbols and filled symbols, respectively. For a deterministic system, Λ is close to 1. For a stochastic system, Λ is much less than 1 and near 0. (a) PCSM2, $N = 807$. Embedding dimension $m = 3$, and resolution = 1/512. (b) DLSM2, $N = 807$. Embedding dimension $m = 3$, and resolution = 1/8.

Appendix B. Testing Correlation Resonances in Searching for Chaos

In Chapter 1, Professor Prigogine pointed out that complex spectral theory plays a critical role in dealing with unstable dynamic systems. Of special importance is the concept of correlation resonances (Ruelle, 1986) which are the peaks of correlation spectra of time series data. We call this approach "the complex spectral analysis of correlations (CSAC)."

A complex Fourier spectrum is a natural generalization of conventional Fourier spectrum when unstable states are present. The complex spectral representation in physics originated from the problem of decaying states and resonances in quantum mechanics (Gel'fand and Vilenkin, 1964). The CSAC shifts interest from the original time series to the correlation functions. Resonances in Henon map were studied by Isola (1988). The complex spectral theory for nonintegrable large Poincare system developed by Petrosky and Prigogine (1991) has been applied to studies of highly chaotic maps by Hasegawa and Saphir (1992). These developments form the foundation for empirical applications of CSAC approach.

Zhang, Wen, and Chen (1992) have improved the CSAC numerical technique for empirical testing with limited data points. The procedure is to first calculate autocorrelations of the time series data, then calculate the power spectra and locate complex singularities of correlations by means of the Pade rational function approximation, and, finally, estimate resonance frequencies and their exponential decay rates.

We tested several log-linear detrended data, including S&P 500, Federal Reserve M2, crude oil prices, and real GNP data. Our initial results, which are reported in Table B.1, provide strong evidence of economic chaos in monetary data, and weak evidence in stock market and oil price data. Weaker evidence is provided in the case of GNP.

The CSAC approach is just one possible test that should be added to the package of numerical tests briefly outlined in this chapter. Each test may reveal one or two aspects of nonlinearity and long-range correlation. The tests are complementary and should all be used in a weighted judgment.

The empirical findings shed important light on endogenous economic dynamics. First, the generalized concept of unstable periodic modes breaks the intellectual barrier between chaos and noise— the two idealized models of complex reality. Now we may define the peaks (resonances in correlation power spectra) with exponential decays as one of many operational indicators of typical chaos.

Second, the existence of correlation resonances in economies builds a link between nonequilibrium evolutionary processes and static equilibrium representations. The correlation resonances reveal the coexistence of various characteristic fluctuations. The lifetime of metastable modes provides a quantitative measure of the transition from disequilibrium to equilibrium.

Table 15.2 Testing Economic Chaos Based on Various Methods

Name	LFDS&P	S&P	M2	GNP	Brent
N-points	1499	1500	807	164	580
Period	1952–81	1952–81	1969–84	1946–88	1983–85
Time unit (Δ)	5 days	5days	week	quarter	day
$T_d(\Delta)$	2	136	61	22	44
$\lambda(\Delta^{-1})$	0.014	0.0145	0.013	0.032	0.020
$\lambda^{-1}(\Delta)$	71.4	69	74.1	31	50
D	no	2.0	1.5	2.5(?)	1.5
P-Portrait	random	spiral	spiral	spiral	spiral
KG Test	no	no	yes	?	no
T_1	no	43.8 ys[?]	4.39ys	33.4ys	140.9 ds
τ_1		43.9 ys[?]	2.44 ys	*	55.7 ds
T_2	no	3.33 ys	1.98 ys	12.5 ys	24.2 ds
τ_2		13.0 ys	0.91 ys	?	47.7 ds
T_3	no	1.09 ys		5.6 ys	18.5 ds
τ_3		*		?	20.6 ds
chaos evidence	no	weak	strong	weaker	weak

[a] Where λ is Lyapunov exponent, T_d is decorrelation time measured from the first zero of autocorrelations, *D* i tion dimension, *P*-portrait represents phase portrait, and KG stands for Kaplan-Glass direct test. T $(=2\pi/\omega)$ is t of correlation resonance, $\tau(=1/\alpha)$ is the lifetime of unstable mode $e^{-\alpha t}e^{i\omega t}$.
Note: The question mark (?) casts doubt on the numerical reliability when data are short. The asterisk (*) indi the extremely slow decay actually means persistent oscillation within the numerical precision.

The results of the observed resonance frequencies are consistent with th mon experience of business cycles (Gordon, 1986). The different rates of ex tial decay provide good indicators of adjustment speed for persistent mediur ness cycles and faster dissipative innovation shocks.

Note

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who has pioneered the use of nonlinear dynamics in explaining complex phys-ical and chemical processes, who recognized very early the relevance of non-linear methods for understanding the wider world of biological and human pro-cesses, and whose leadership, example, and personal support have encouraged others to develop social science in this promising direction.